Statistical Physics of Quantum Matter 2013

Universal quantum computation with AKLT states

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In contrast to the standard circuit model, quantum computation can actually be achieved by performing single-spin measurements on certain entangled states. Although there is still a lack of a complete classification for such resourceful entangled states, a few states are known to provide universal quantum computation. These include, among others, the original cluster states on regular lattices and more recently a spin-$\frac{3}{2}$ AKLT state–a valence-bond solid ground state—on the honeycomb lattice. However, it is unclear whether with regards to quantum computation the spin-$\frac{3}{2}$ AKLT state is a special case in the family of AKLT states, which can be disordered, Neel ordered, or even spin liquid, depending on the underlying lattices. Here, we shall explain why the spin-$\frac{3}{2}$ AKLT state is a universal resource for the measurement-based quantum computation and how it hinges on the notion of percolation from statistical mechanics. We shall show that the spin-$\frac{3}{2}$ AKLT states on other trivalent lattices are also universal and moreover so is a mixture of spin-$\frac{3}{2}$ and spin-2 AKLT state. Our approach may enable investigation of higher-spin AKLT states for quantum computation, including the spin-2 case on the square and Kagome lattices. The importance of AKLT states for quantum computation also comes from the uniqueness of the ground state (for a suitable boundary condition) and the very likely existence of a spectral gap above the ground state.