UNIVERSITÄT DES SAARLANDES

The 2d Disordered Bose-Hubbard Model: Phase Diagrams and New Applications

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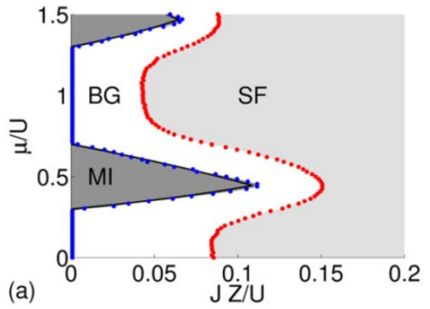
Statistical Physics of Quantum Matter, Taipei, 28.-31.7.2013

Outline

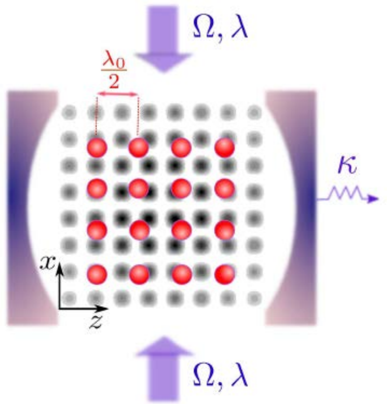
Part 1



Superfluid Clusters, Percolation and Phase transitions
in the Disordered, Two-Dimensional Bose–Hubbard Model

w. Astrid Niederle

Part 2



Bose-Glass Phases of Ultracold Atoms due to Cavity Backaction

w. André Winter, Hessam Habibbian,
Simone Paganelli, Giovanna Morigi

The disordered Bose-Hubbard model (BHM)

$$H = -J \sum_{(i,j)\ n.n.} \hat{a}_i^+ \hat{a}_j + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) - \sum_i (\mu - \epsilon_i)\, \hat{n}_i$$

$\hat{a}_i$ Boson operators, $[\hat{a}_i^+, \hat{a}_i] = \delta_{ij}$

$\hat{n}_i = \hat{a}_i^+ \hat{a}_i$ particle number operatore (at site i)

J hopping strength

U onsite repulsion

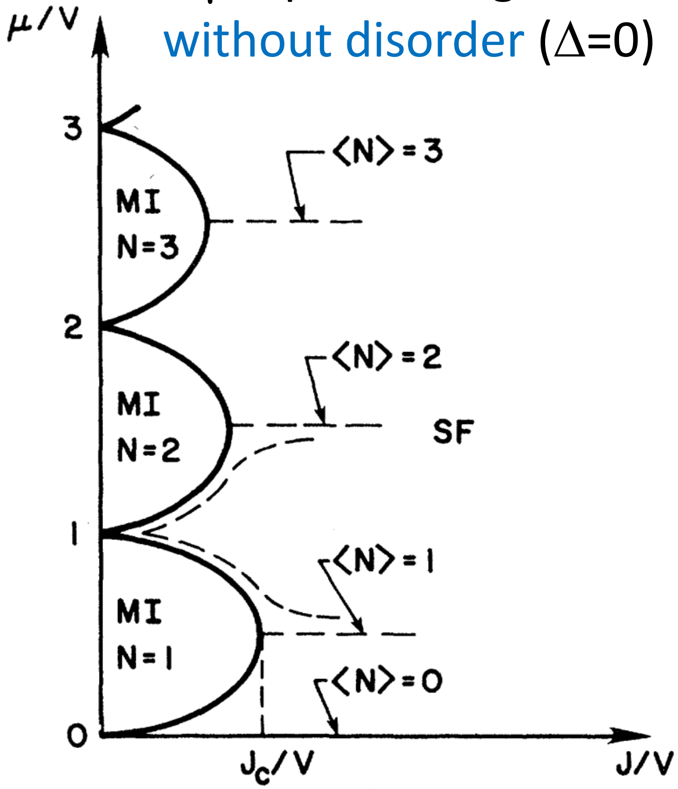
μ chemical potential

ε_i random on-site energy, e.g. $\varepsilon_i \in [-\Delta/2, +\Delta/2]$

- Originally introduced to describe phase transitions in superfluids with quenched disorder (e.g. **He^4 in aerogels**)
- Renewed interest motivated by **ultra-cold atoms** in (disordered) optical lattices

The putative phase diagram

μ/J phase diagram
without disorder (Δ=0)



MI: Mott insulator (ρ_{SF}=0, κ=0)

SF: Superfluid (ρ_{SF}>0),
compressible (κ>0)
(gapless)

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Boson localization and the superfluid-insulator transition

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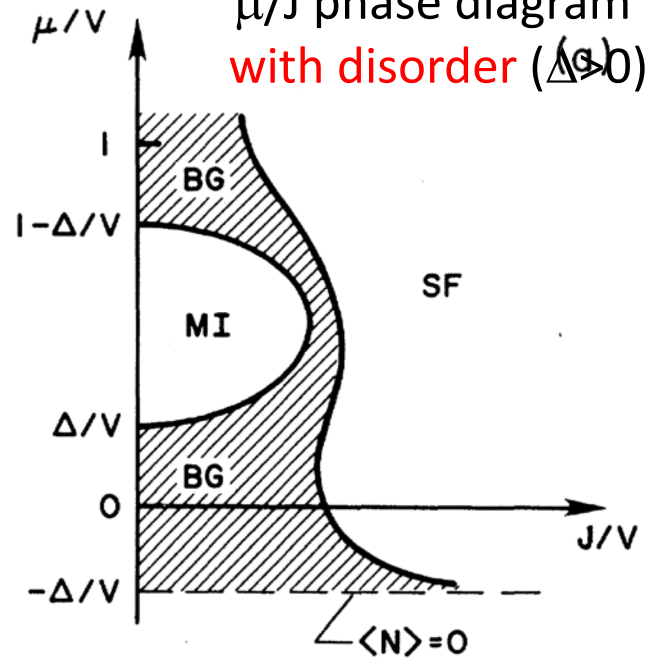
(Received 15 November 1988)

μ/J phase diagram
with disorder (Δ>0)



BG: **Bose glass** (ρ_{SF}=0, κ>0)
i.e. non-SF, but compressible (gapless)

What is a Bose glass? Excursion to RTFIM …

Reminder: random transverse field Ising model (RTFIM)

$$H = -\sum_{(i,j)\ n.n.} J_{ij}\hat{\sigma}_i^z\hat{\sigma}_j^z + h\sum_i \hat{\sigma}_i^x$$

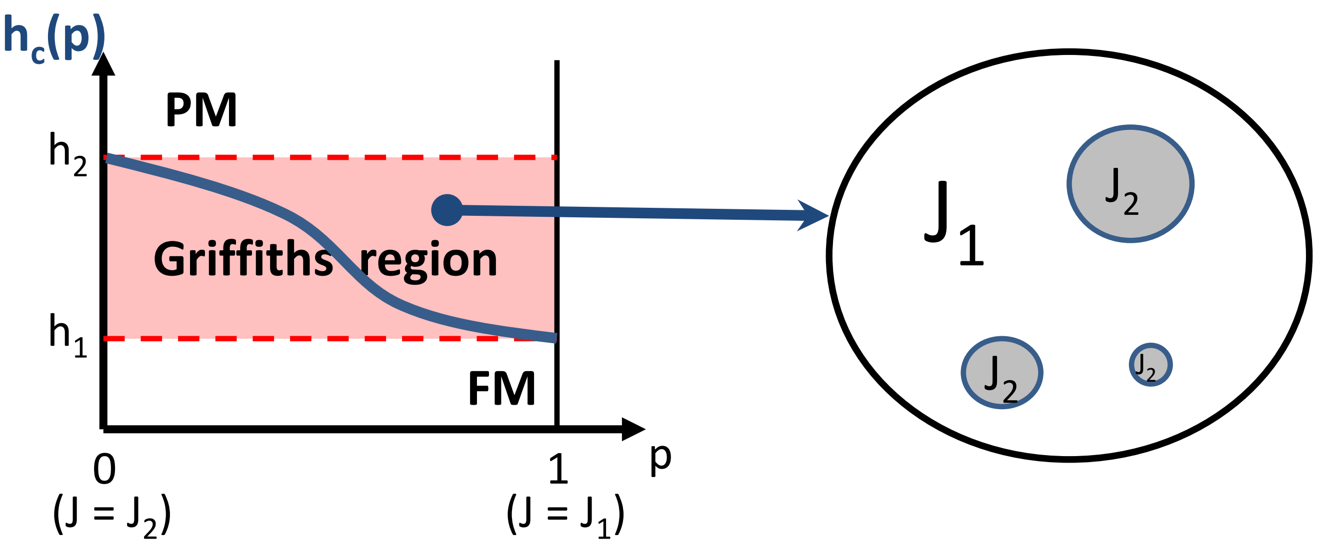
$\hat{\sigma}_i^{x,y,z}$ Spin-1/2 operators, $[\hat{\sigma}_i^x, \hat{\sigma}_j^y] = i\hat{\sigma}_i^z \cdot \delta_{ij}$

h transverse field strength

J_{ij} random ferromagnetic couplings

Consider binary disorder: $J_{ij} = J_1$ with prob. p

$J_{ij} = J_2$ with prob. 1-p $\quad J_1 < J_2$



FM clusters
rare regions
small gaps
large relaxation times

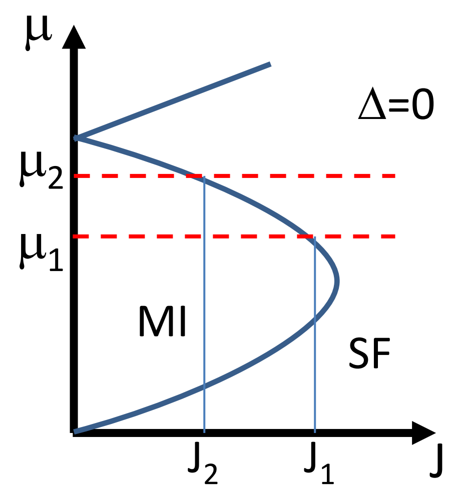
⇒ algebraic singularities

(c.f. talk of F. Iglói)

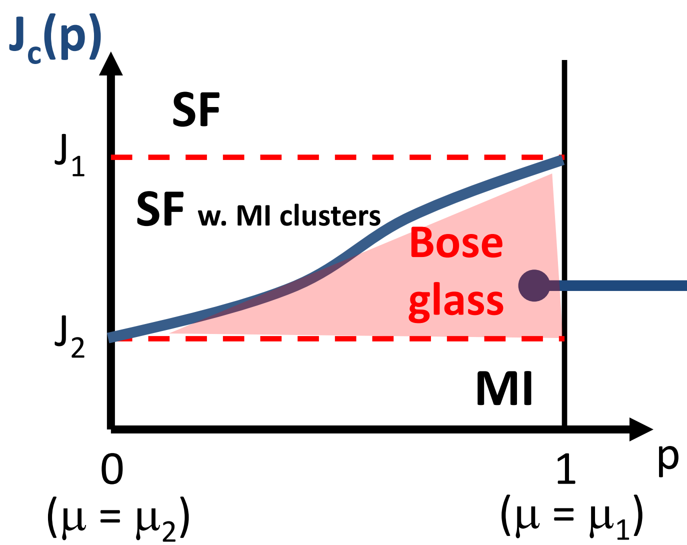
Bose glass = Griffiths phase of disordered BHM

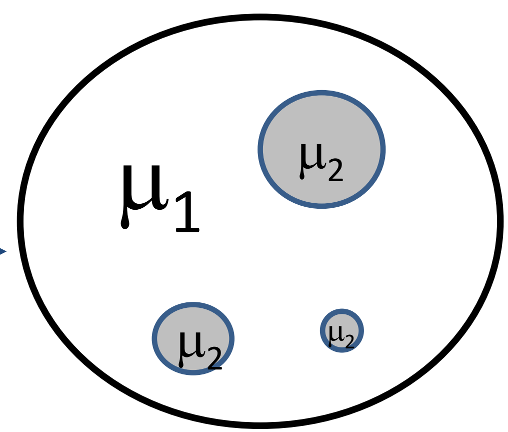
$$H = -J \sum_{(i,j)\ n.n.} \hat{a}_i^{+} \hat{a}_j + \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1) - \sum_i \mu_i \hat{n}_i$$

Consider binary disorder: $\mu_i = \mu_1$ with prob. p
$\mu_i = \mu_2$ with prob. 1-p $\quad \mu_1 < \mu_2$



Let $J_c(p)$ be the critical hopping strength for SF,
i.e. $J > J_c(p) \Rightarrow \rho_{SF} > 0$
$J < J_c(p) \Rightarrow \rho_{SF} = 0$



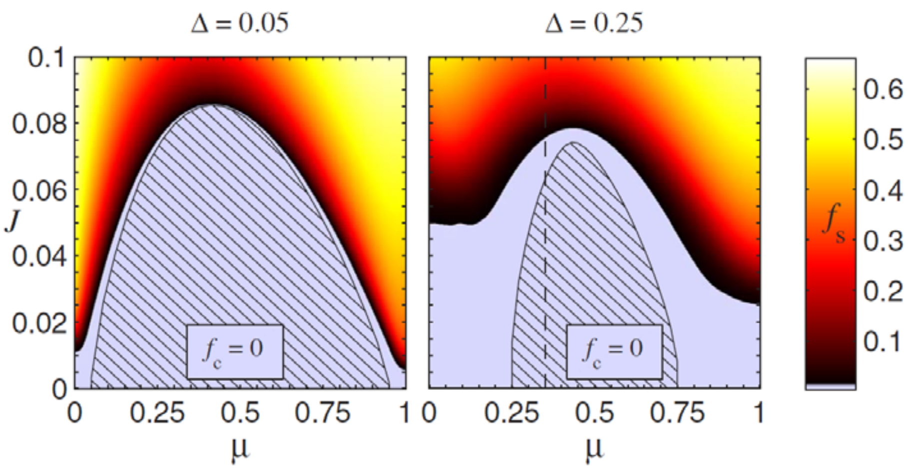


SF clusters
rare regions
small gaps
$\Rightarrow$ singularities (not algebraic, because of cont. symmetry)

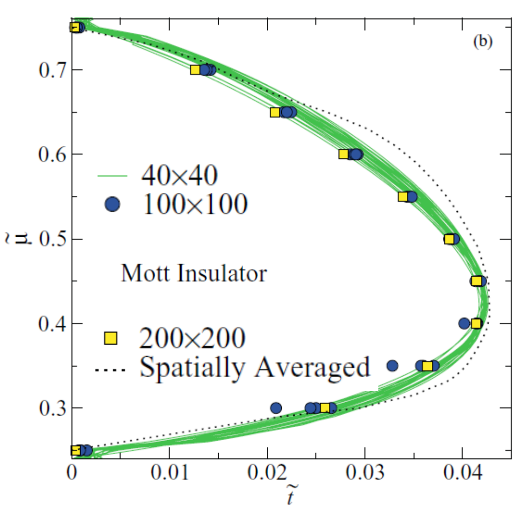
n.b.: SF clusters $\Rightarrow$ **no direct SF-MI transition**
(for rigorous treatment see Pollet et al, 2009)

Various predictions for phase diagram with fixed Δ

Local MFT, computation of stiffness



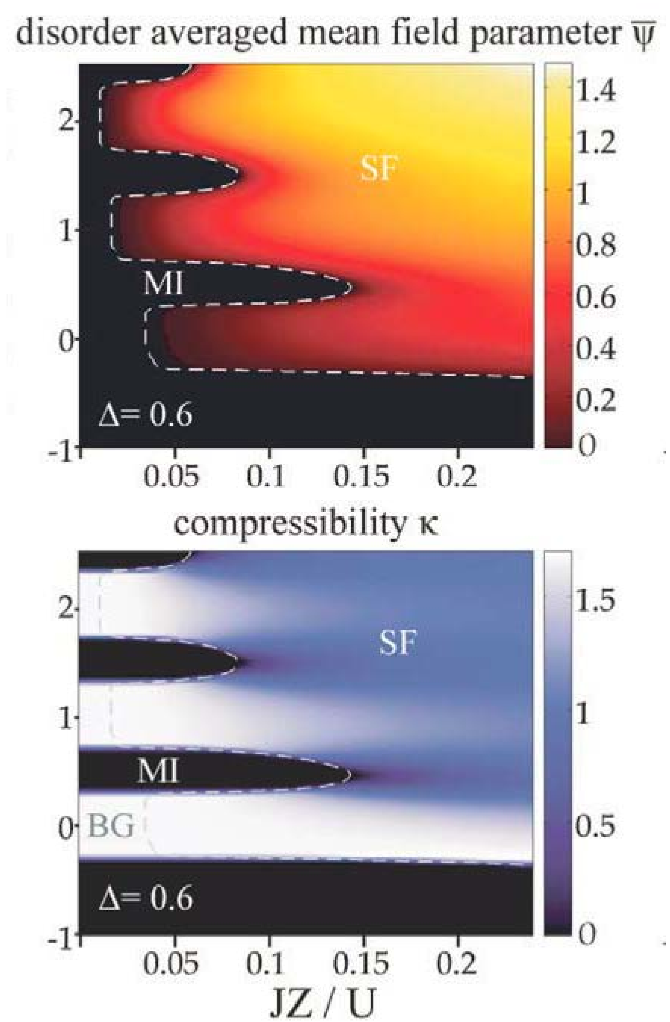
Buonsante et al, PRA 76, 011602 (2007)



Multistite MFT

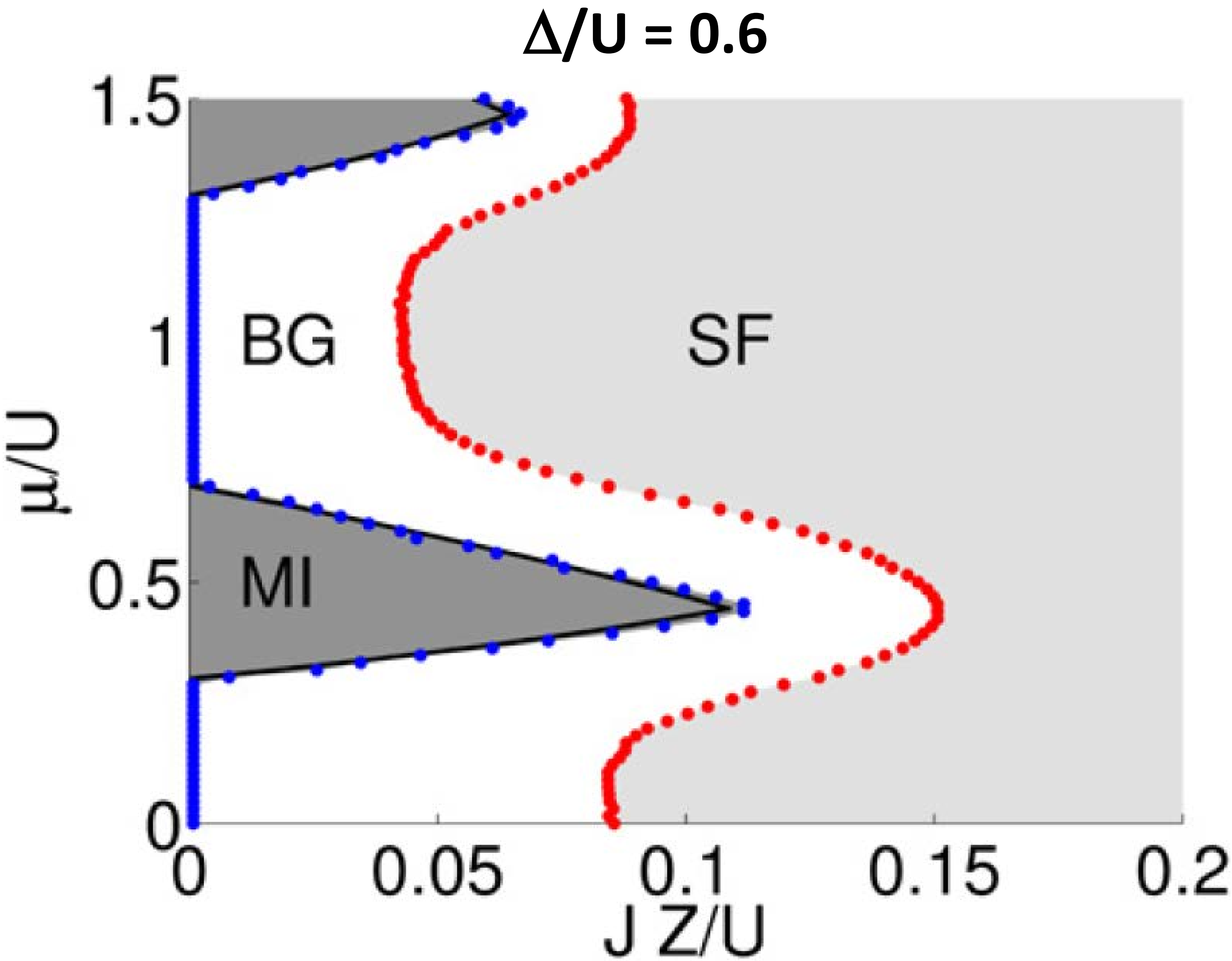
Pisarski et al,
PRA 83, 053608
(2011)

Stochastic MFT



Hofstetter et al, EPL 86, 50007 (2009)

How phase diagram should look (for fixed Δ, in 2d):



[A. Niederle, HR, NJP (2013)]

Identification of SF / BG / MI phase in d ≥ 2

via global observables:

SF: superfluid fraction or stiffness $\rho_s = \lim_{\theta\to 0} \frac{E_\theta - E_0}{\langle \hat{N} \rangle J \theta^2} > 0$

compressibility $\kappa = \langle \hat{N}^2 \rangle - \langle \hat{N} \rangle^2 > 0$

BG: $\rho_s=0$, $\kappa>0$

MI: $\rho_s=0$, $\kappa=0$

via local occupation number:

Def.: $S_i = \begin{cases} 0 & \text{if } \langle \hat{n}_i \rangle \text{ integer} \\ 1 & \text{else} \end{cases}$

Def.: **SF-cluster**:

connected cluster with $\forall_i$ $S_i=1$

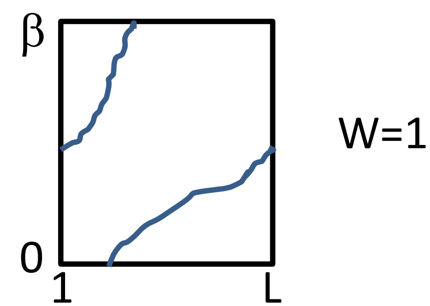
n.b.: $\langle n_i \rangle$ non-integer ⇔ $\langle a_i \rangle \neq 0$

SF: at least one SF-cluster percolates
BG: SF-clusters exist, but none percolates
MI: no SF-cluster exist s

[A. Niederle, HR, NJP (2013)]

Motivation:

(1) World line QMC: $\rho_s \sim \langle W^2 \rangle$,
W = winding number



(2) mappping to quantum rotors
↔ (d+1)-dim XY-like model

Local Mean Field Theory (LMFT)

$$H = -J \sum_{(i,j)\ n.n.} \hat{a}_i^{+} \hat{a}_j + \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1) - \sum_i \mu_i \hat{n}_i$$

Approximate hopping term: $\hat{a}_i \hat{a}_j^{\dagger} \approx \hat{a}_i \langle \hat{a}_j^{\dagger} \rangle + \hat{a}_j^{\dagger} \langle \hat{a}_i \rangle - \langle \hat{a}_i \rangle \langle \hat{a}_j^{\dagger} \rangle$

$\rightarrow H_{LMF} = \sum_i H_i$ with $\hat{H}_i = (\epsilon_i - \mu)\, \hat{n}_i + \frac{U}{2} \hat{n}_i \left(\hat{n}_i - 1 \right) - J\, \eta_i \left(\hat{a}_i + \hat{a}_i^{\dagger} - \psi_i \right)$

$$\eta_i = \sum_{j \text{ n.n. of } i} \psi_j$$

and $\psi_i = \langle \hat{a}_i \rangle = \langle \Psi | a_i | \Psi \rangle$ the local SF parameter
to be determined self-consistently

GS $|\Psi>$ of H_{LMF} is a Gutzwiller state: $|\Psi\rangle = \prod_{i=1}^{M} \left(\sum_{n=0}^{\infty} c_n^i |n\rangle_i \right)$

Solve self-consistency equations for $\{\psi_i\}$ numerically,
Calculate average SF order parameter, compressibility, etc.

Problems of Averaged Order Parameter / Compressibility

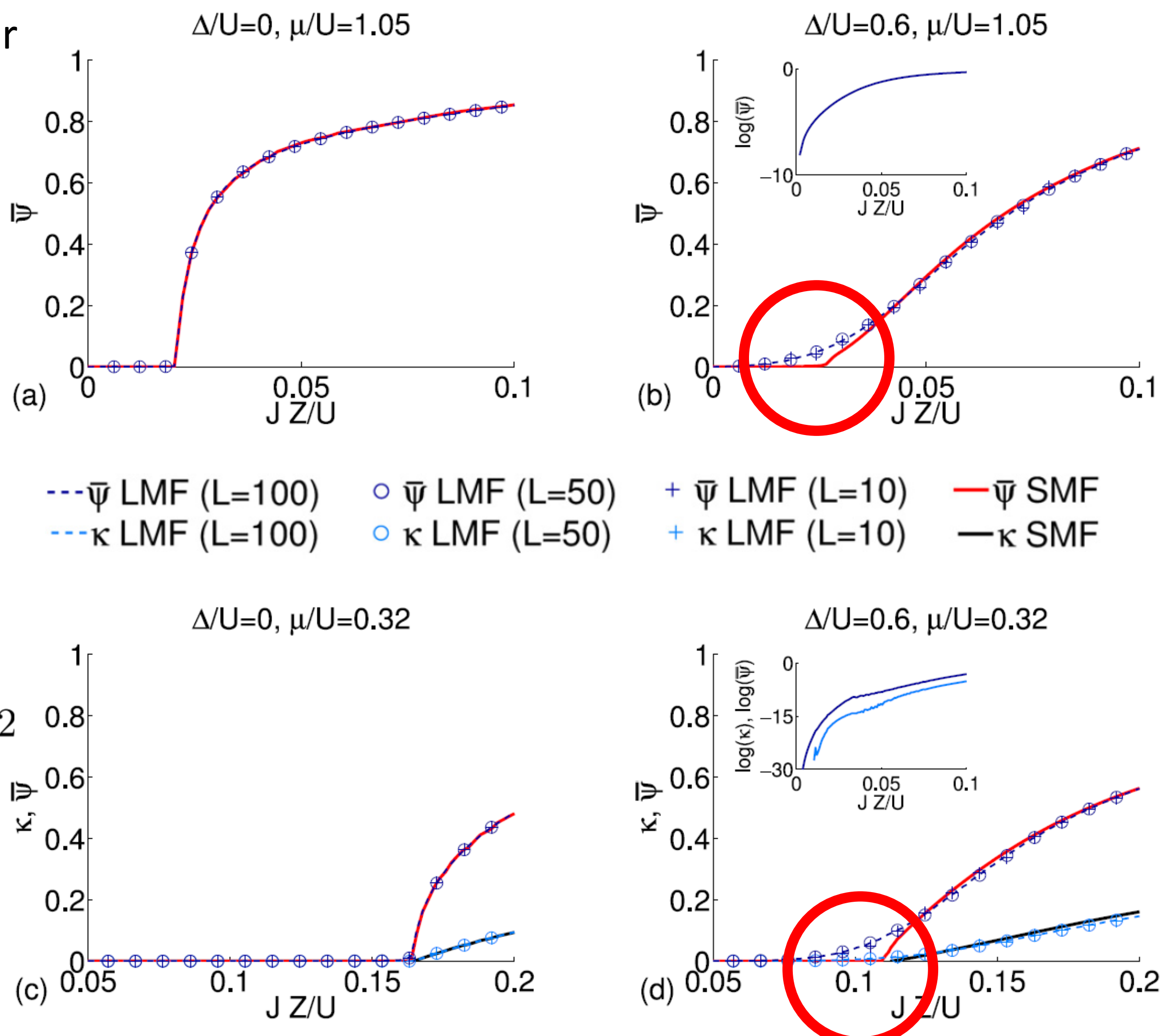
No disorder | Disorder

average SF parameter

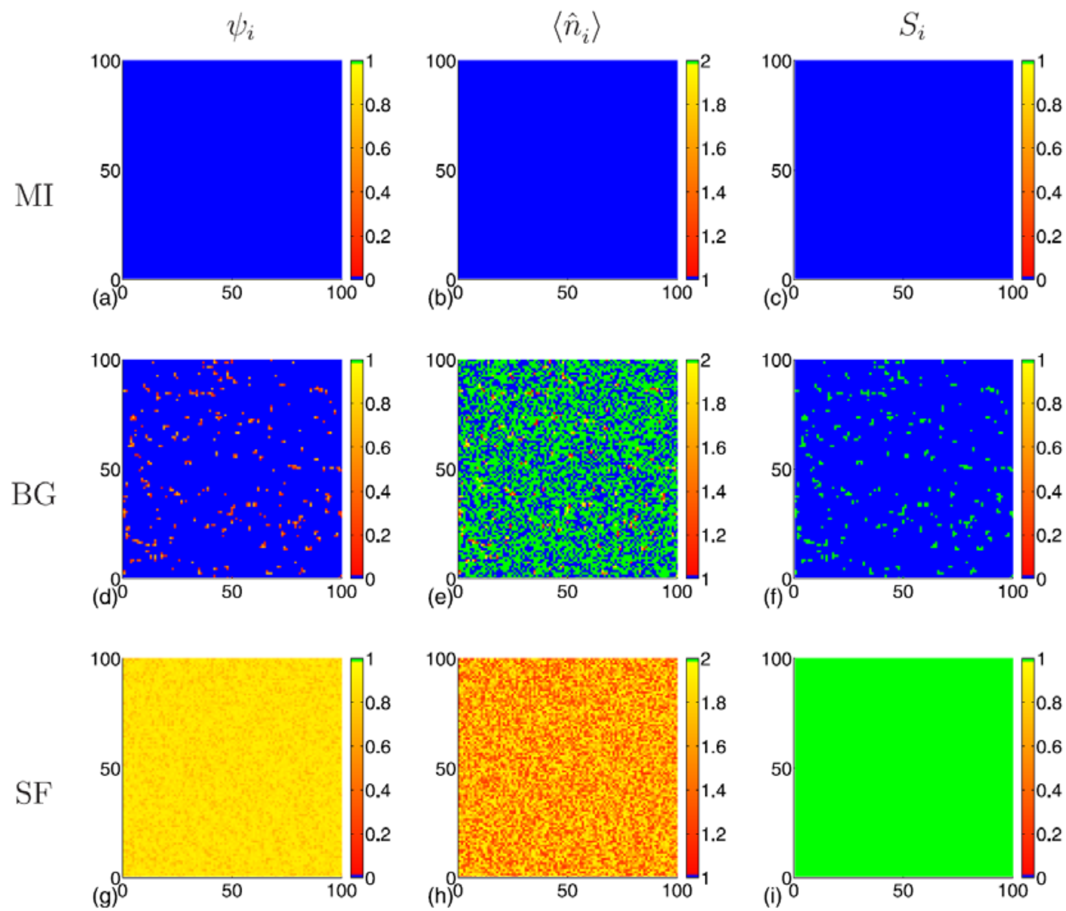
$$\overline{\Psi} = \frac{1}{N}\sum_{i=1}^{N}\langle \hat{a}_i \rangle$$

compressibilty

$$\kappa = \langle \hat{N}^2 \rangle - \langle \hat{N} \rangle^2$$



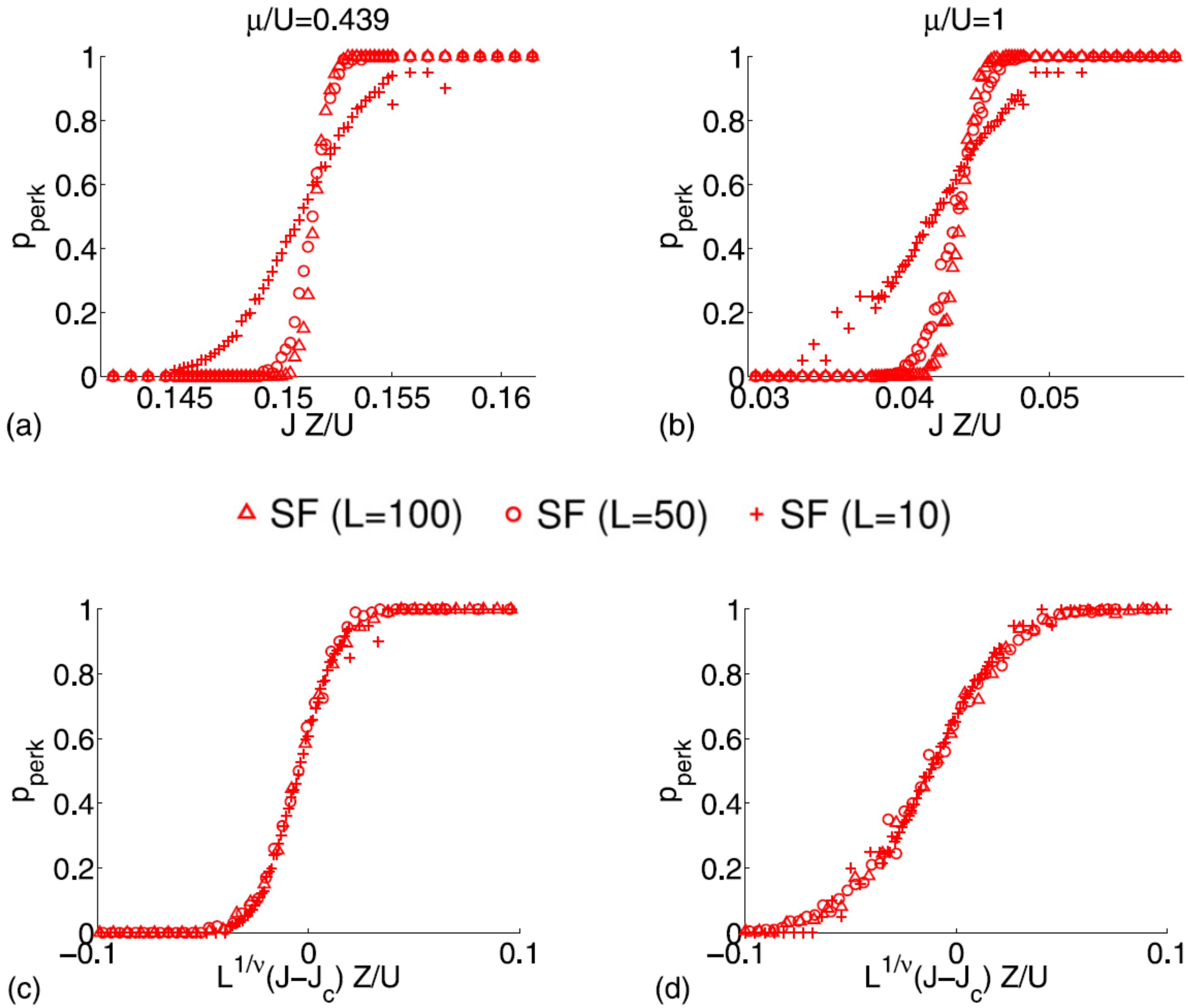
SF-clusters in the different phases



$$S_i = \begin{cases} 0 & \text{if} \quad I - \gamma_n \leqslant \langle \hat{n}_i \rangle \leqslant I + \gamma_n, \qquad I = 0, 1, 2, \ldots, \\ 1 & \text{else}, \end{cases}$$

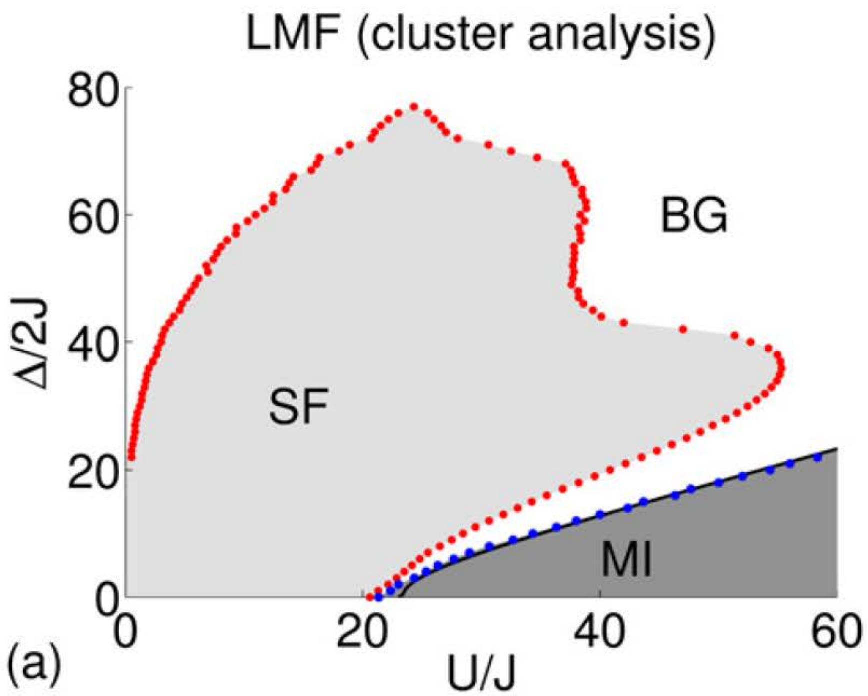
$$\gamma_n = 5 \times 10^{-3}$$

Percolation transition / Finite Size Scaling



$\upsilon = 4/3$

Phase diagram for fixed density ρ=1



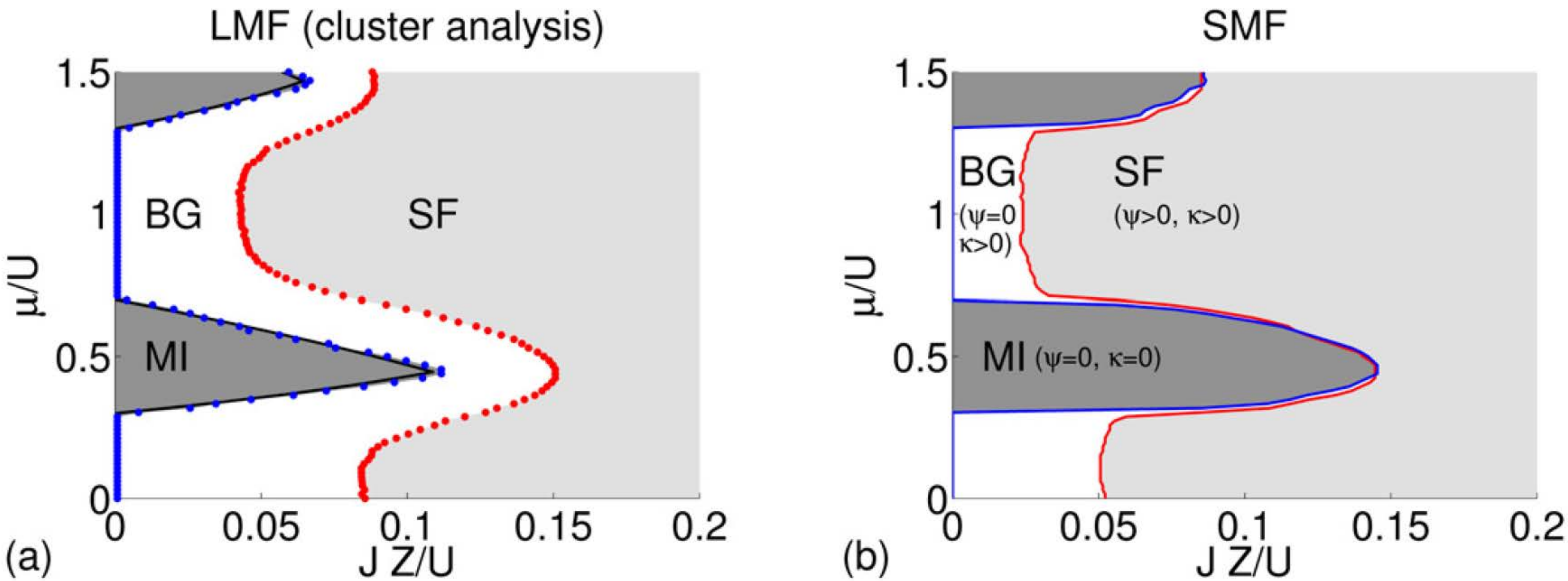
[A. Niederle, HR, NJP (2013)]

red dots: quantum Monte Carlo results
Söyler et al, PRL 107, 185301 (2011)

blue dots: gap data for pure system $E_{g/2}=\Delta/J$

red broken line: Falco et al. (2009)

Phase diagam for fixed disorder (Δ/U=0.6)



[A. Niederle, HR, NJP (2013)]

[Hofstetter et al, EPL 86, 50007 (2009)]

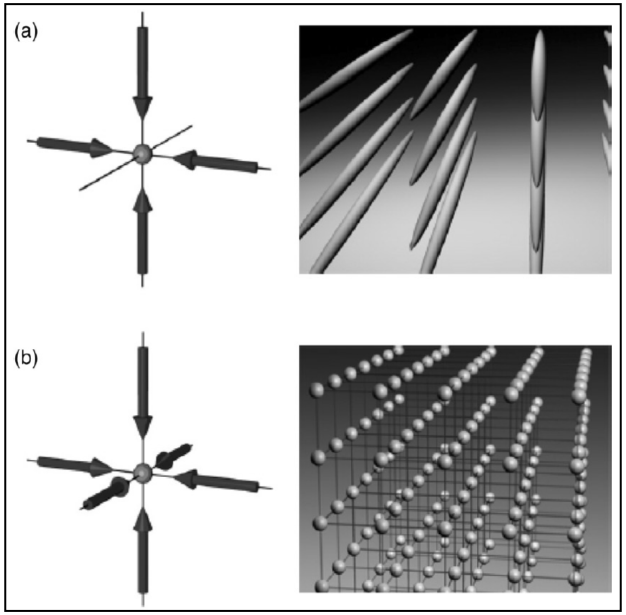
n.b.: stochastic MFT calculates
$P(\psi)$ self-consistently,
assuming that $P(\psi_i)$ is identical $\forall_i$
$\Rightarrow$ neglects spatial inhomogeneities

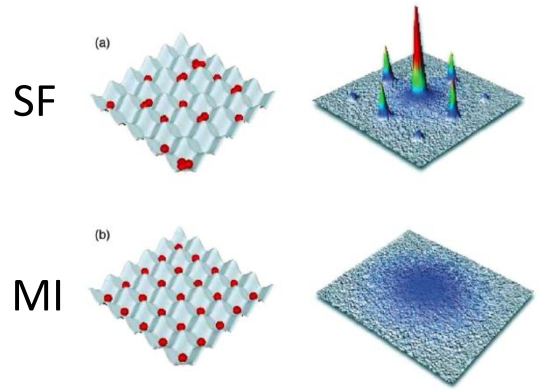
Conclusion 1

- SF-cluster analysis yields good estimate of phase diagram for d=2, 3 using LMFT
- Fast and easy method (for disordered / aperiodic BHM in $d \geq 2$)
- Hypothesis: BG-SF transition is a percolation transition – check with QMC
- Does not work in d=1
- Binary disorder: SF-cluster percolation $\neq$ disorder cluster percolation

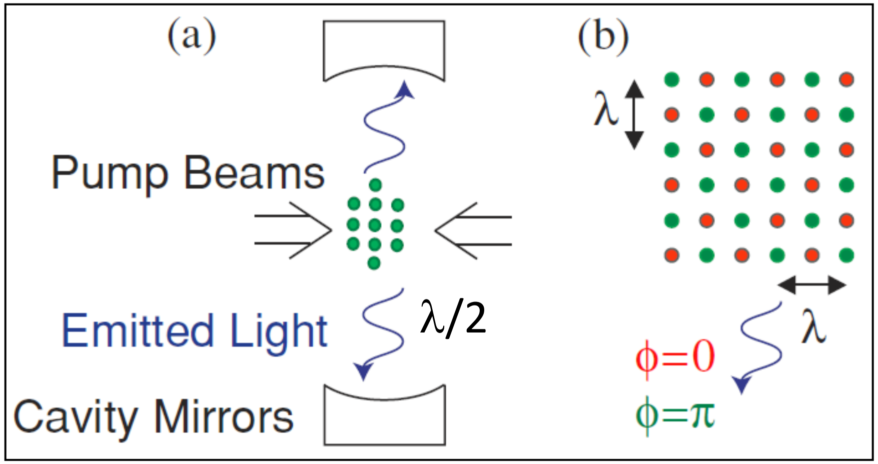
Optical lattices vs. self-organization of cold atoms

Optical lattices



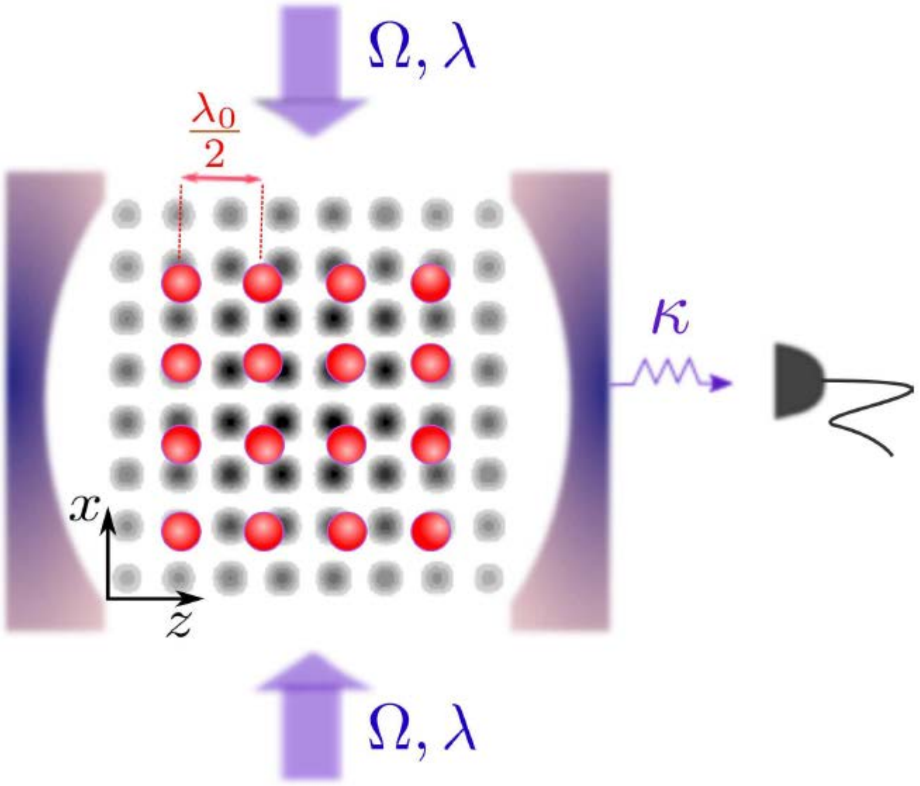


Collective spatial self-organization of two-level atoms and emitted light



Theory: Domokos, Ritsch, PRL 89, 253003 (2002)
Exp.: Black, Chan, Vuletic, PRL 91, 203001 (2003)

Bose Glass phase due to Cavity Backaction



- Ultra-cold atoms in optical lattice, lattice constant λ_0
- put in a cavity in z-direction
- add a pump laser in x-direction, wave length $\lambda/2$
- λ and λ_0 incommensurate

Effective Hamiltonian for the atoms: 2d BHM

$$H = -J \sum_{(i,j)\ n.n.} \hat{a}_i^+ \hat{a}_j + \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1) + \sum_i (\mu_0 + \delta\hat{\mu}_i)\,\hat{n}_i$$

$$\delta\hat{\mu}_i = -V_1 J_0^{(i)} - \hbar \frac{s_0^2}{\hat{\delta}_{\text{eff}}^2 + \kappa^2} \hat{\Phi}(2\hat{\delta}_{\text{eff}} Z_0^{(i)} + u_0 \hat{\Phi} Y_0^{(i)})$$

$$\hat{\delta}_{\text{eff}} = \delta_c - u_0 \sum_i Y_0^i\, \hat{n}_i / L^2$$

$$\hat{\Phi} = \sum_i Z_0^i \hat{n}_i / L^2$$ = cavity field

Φ^2 ~ number of photons in the cavity

n.b.: cavity field induces long range interactions $\propto s_0^2 \sum_{i,j} \hat{n}_i \hat{n}_j$

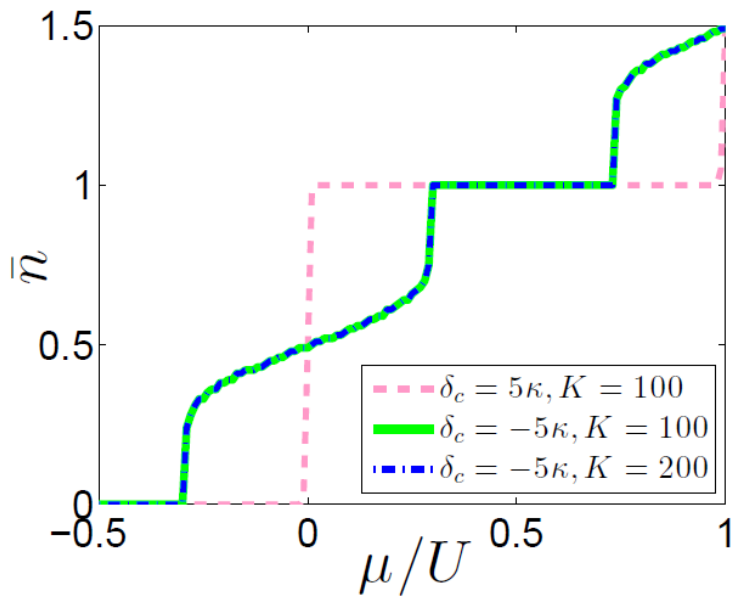
$$J_0^i = \int d^2\mathbf{r}\ \cos^2(kx)\, w_i^2(\mathbf{r})$$

$$Y_0^i = \int d^2\mathbf{r}\ \cos^2(kz)\, w_i^2(\mathbf{r})$$

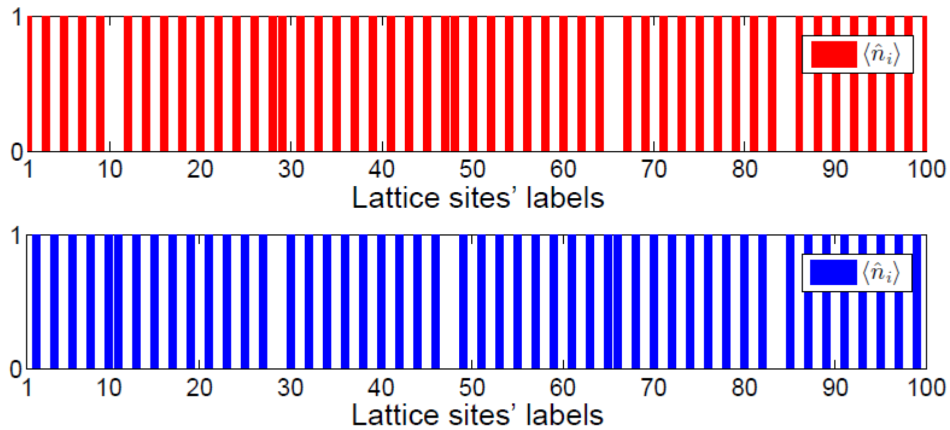
$$Z_0^i = \int d^2\mathbf{r}\ \cos(kz)\cos(kx)\, w_i^2(\mathbf{r})$$

[Habibian, Winter, Paganelli, HR, Morigi: PRL 110, 075304 (2013), arXiv:1306.6898]

Zero hopping limit (J=0) in 1d

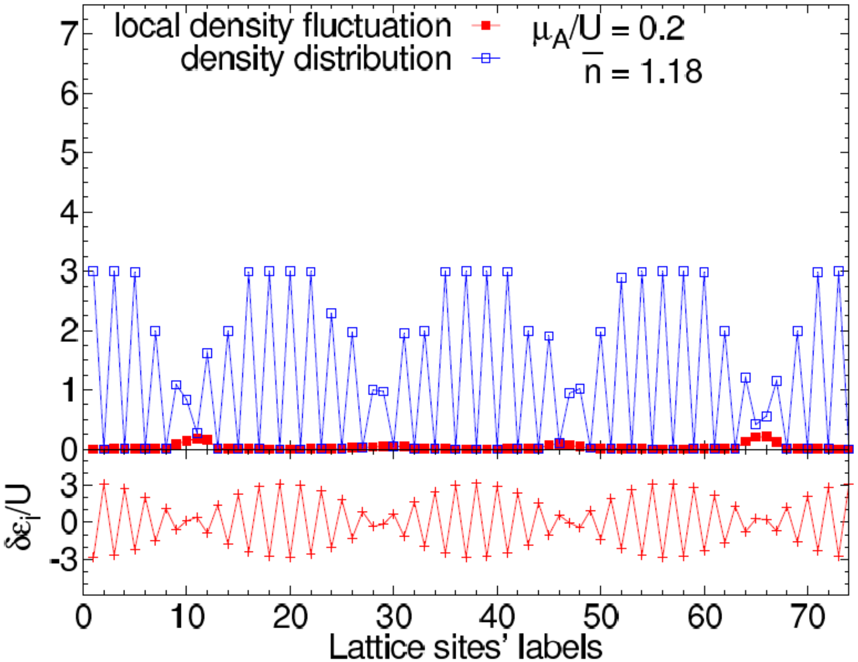


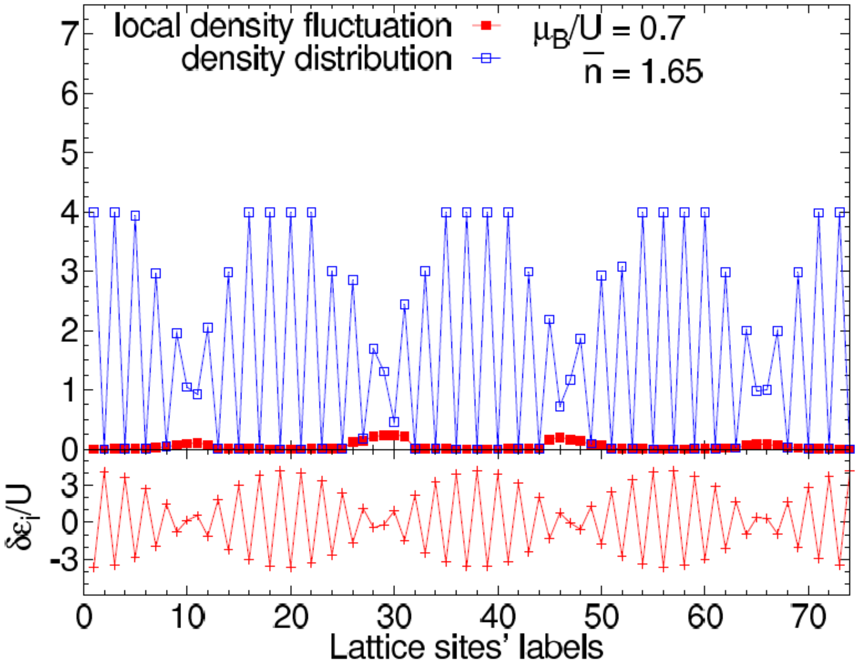
2 ground states



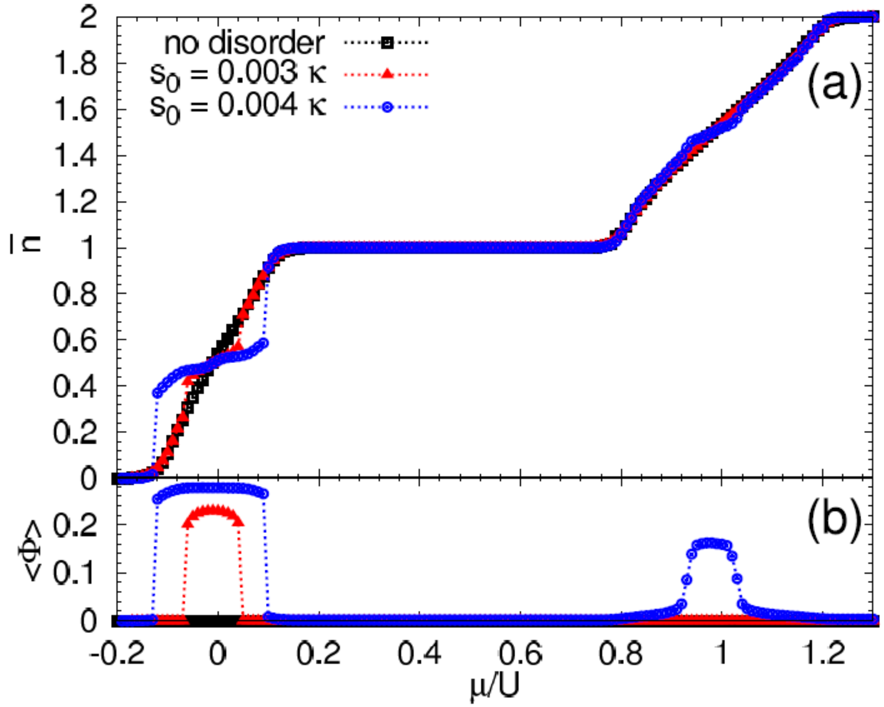
1d, J>0: density oscillations

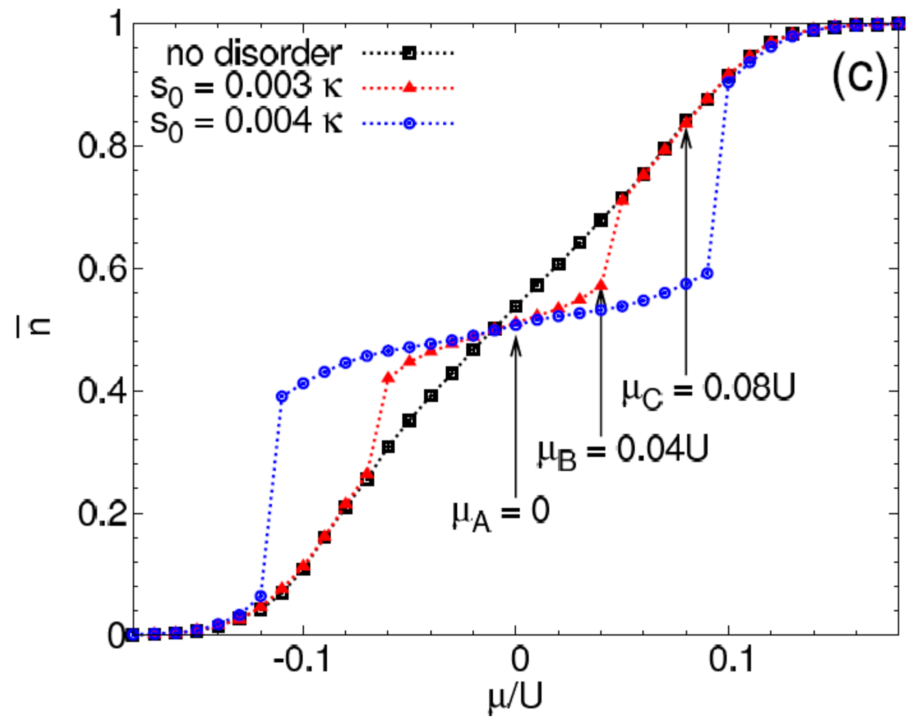
$d/\lambda_0 = 83/157$



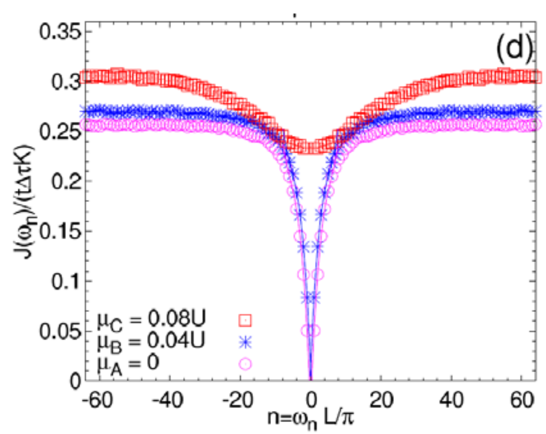


1d, QMC results:

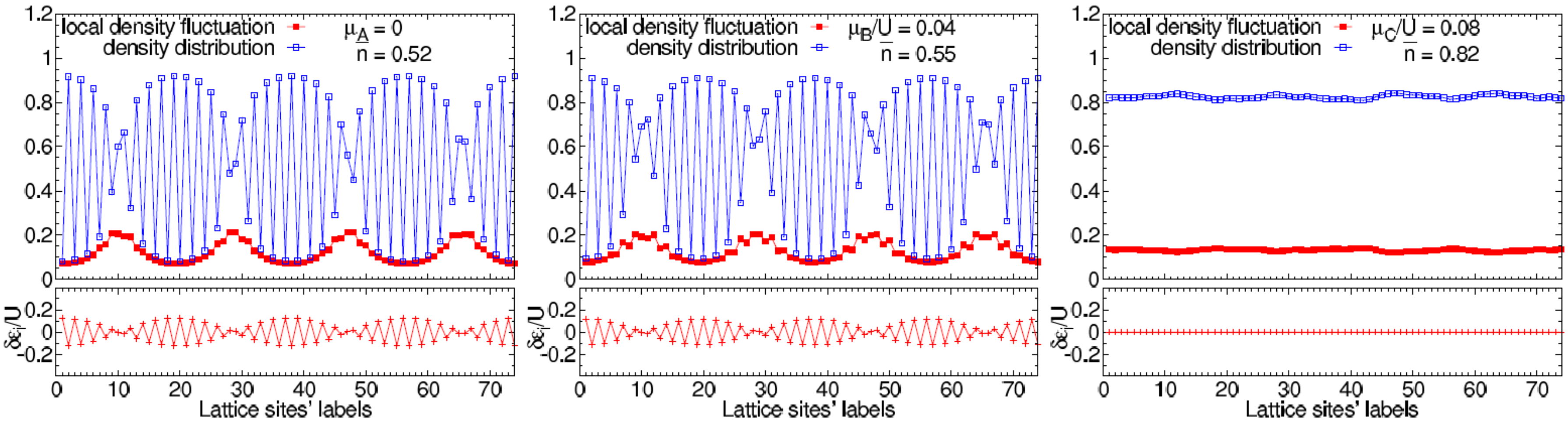


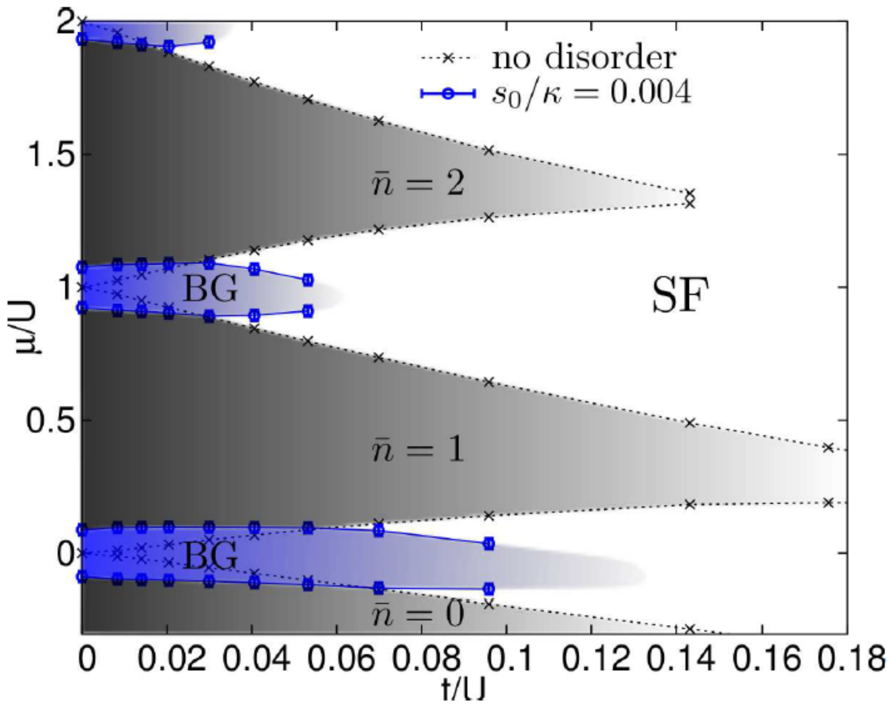


Pseudo current-current correlation function



1d phase diagram (QMC)





[Habibian, Winter, Paganelli, HR, Morigi: PRL 110, 075304 (2013), arXiv:1306.6898]

Conclusion 2

- Similar results in 2d (via LMFT)
- Bose glass phase induced by cavity backaction due to spontaneous emergence of incommensurate potential
- Cavity field induces long range interactions among atoms
- Canonical ensemble and grand-canonical ensemble are equivalent in spite of long range interactions
- direct MI-SF transition (aperiodic potential $\neq$ generic disorder)