The search for a new, self-correcting phase of matter

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Université de Sherbrooke

Statistical Physics of Quantum Matter

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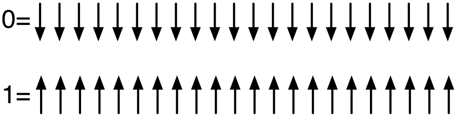
Classical memories are robust



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- Boltzmann: configuration x has probability $\propto \exp(-E(x)/T)$.
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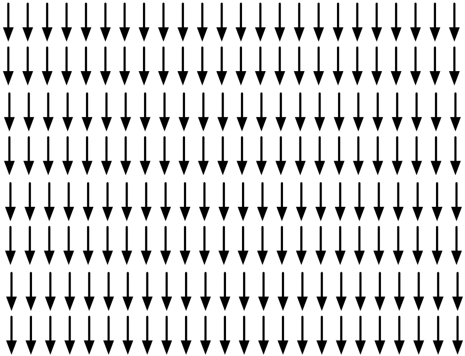

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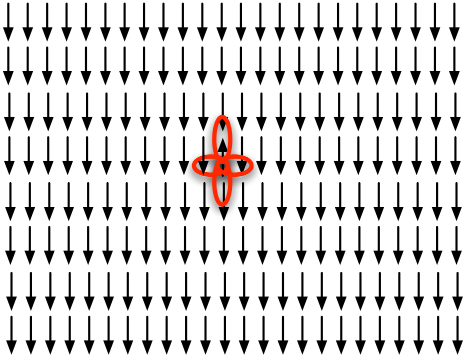

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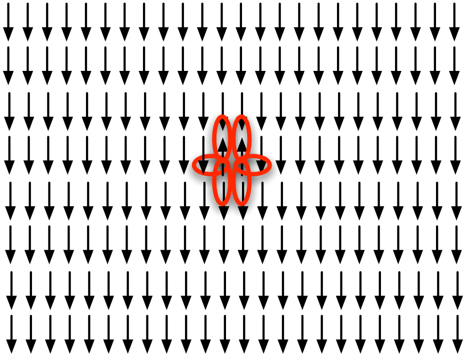

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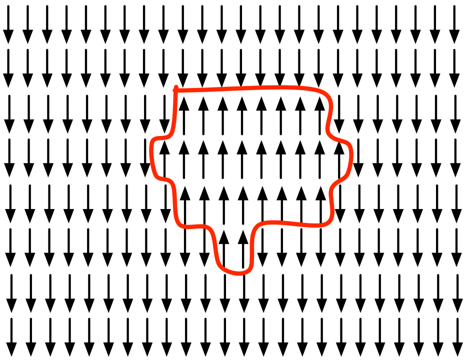

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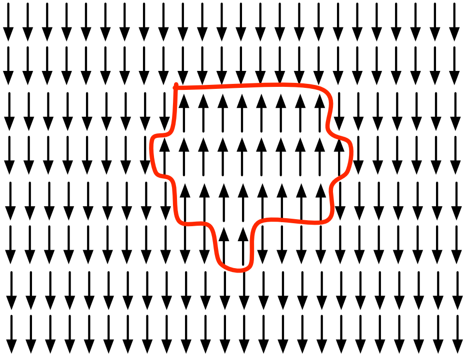

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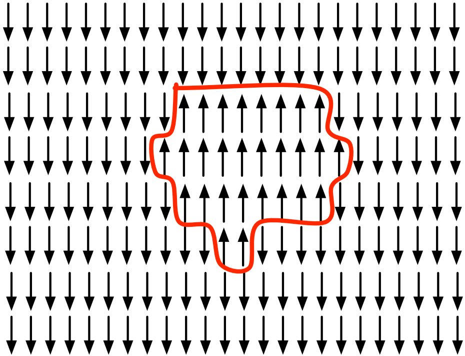

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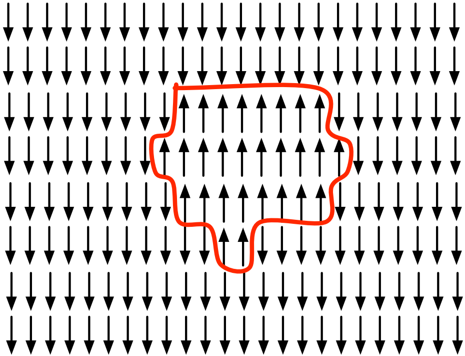

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Local order parameter & decoherence

- System has two ground states $|\uparrow\uparrow\ldots\uparrow\rangle$ and $|\downarrow\downarrow\ldots\downarrow\rangle$.
  - $\alpha|\uparrow\uparrow\ldots\uparrow\rangle + \beta|\downarrow\downarrow\ldots\downarrow\rangle$ does not evolve in time.
- Local observable σ_i^z distinguishes them.
  - Local order parameter σ^z.
- Local perturbation $B\sigma_z$ lifts degeneracy:

$$\alpha|\uparrow\uparrow\ldots\uparrow\rangle + \beta|\downarrow\downarrow\ldots\downarrow\rangle \xrightarrow{t} e^{-iBt}\alpha|\uparrow\uparrow\ldots\uparrow\rangle + e^{iBt}\beta|\downarrow\downarrow\ldots\downarrow\rangle$$

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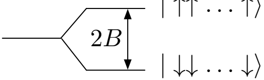

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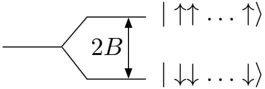

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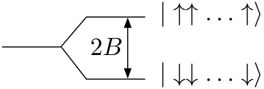

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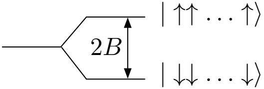

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Bravyi, Hastings, & Michalakis

- (TQO1) System has no local order parameter.
- (TQO2) System is locally consistent.

The system has a stable spectrum.
Long lived memory at zero temperature.

$$H = -\sum_i \sigma_i^z \sigma_{i+1}^z + \sigma_{23}^z$$

The ground state manifold changes abruptly when including site 23.

- Can we combine this spectral stability with the thermal stability of the 2D Ising model?
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- System with degenerate ground state manifold.
- Prepare system in a given ground state $|\psi_g\rangle$.
- Let system evolve in contact with heat bath at temperature T for time t.
- Cool the system to its ground state manifold, and recover a states close to $|\psi_g\rangle$.
- Storage time t scales with system size for $T \leq T_*$.

- Quantum information science & technologies
  - A system that can store quantum information coherently for macroscopic time without active external intervention (quantum hard drive).
- Foundations of physics
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Outline

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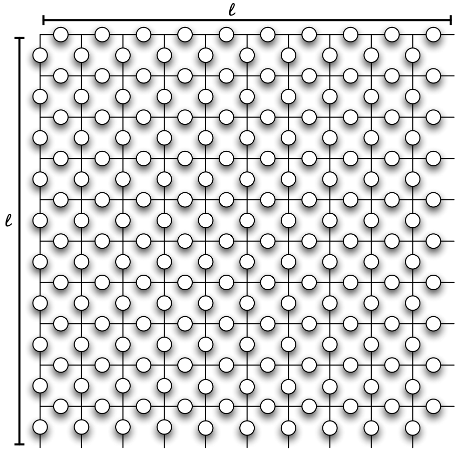
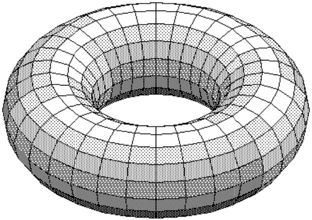


- Two-dimensional square lattice
- Periodic boundary conditions



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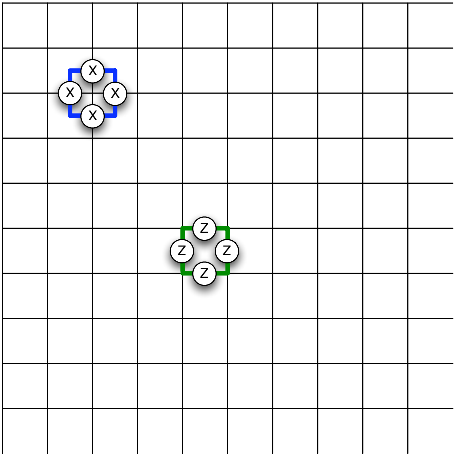

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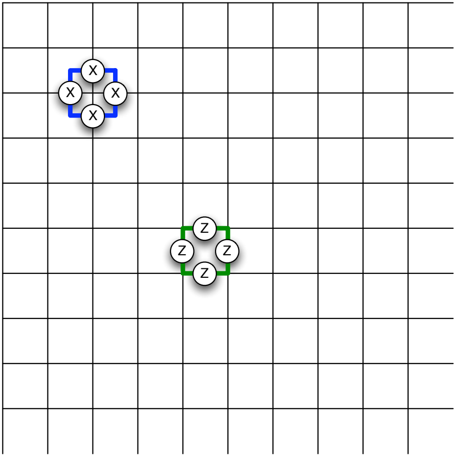

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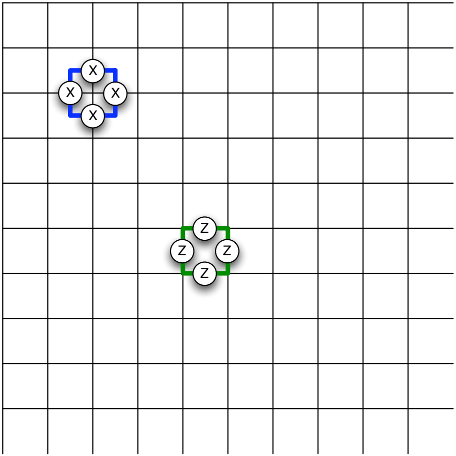

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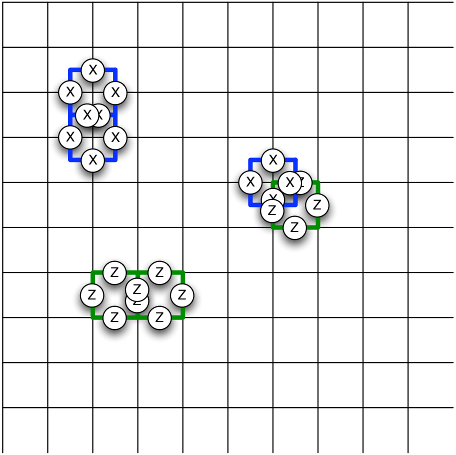

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- The Hamiltonian is a sum of commuting terms.
  - Exactly solvable
  - Constant gap
- Ground space $|\psi\rangle$
  - $A_s|\psi\rangle = +|\psi\rangle$
  - $B_p|\psi\rangle = +|\psi\rangle$

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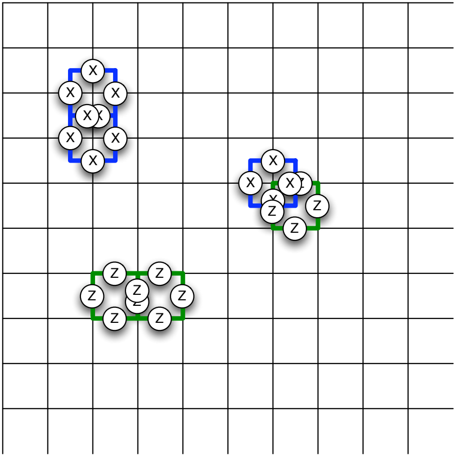

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  - $A_s|\psi\rangle = +|\psi\rangle$
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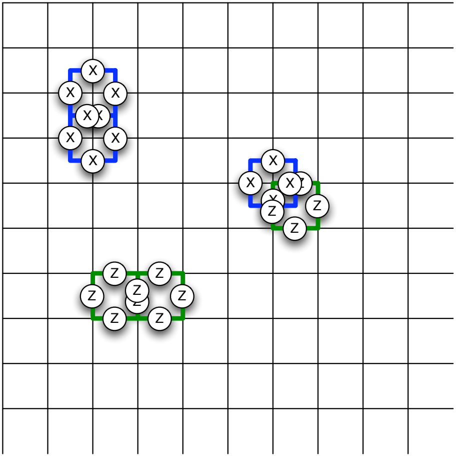
Hamiltonian



- $H = -(\sum_s A_s + \sum_p B_p)$
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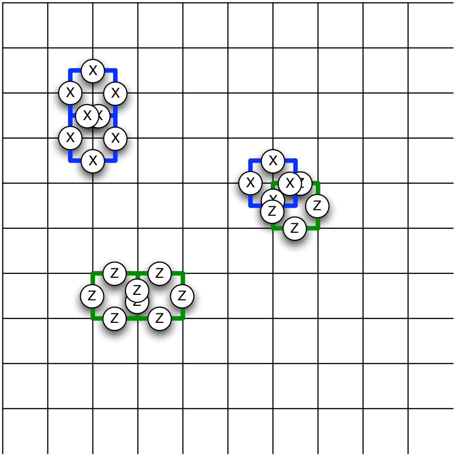

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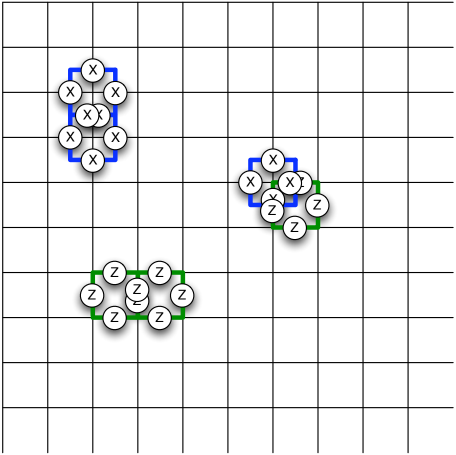

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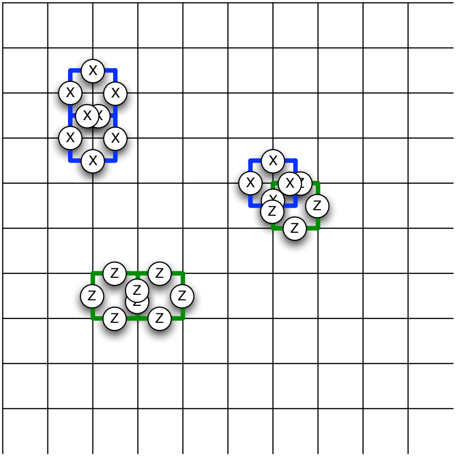

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String operators

- $\overline{Z}_1 = \prod_{i \in \gamma_1} \sigma_z^i$
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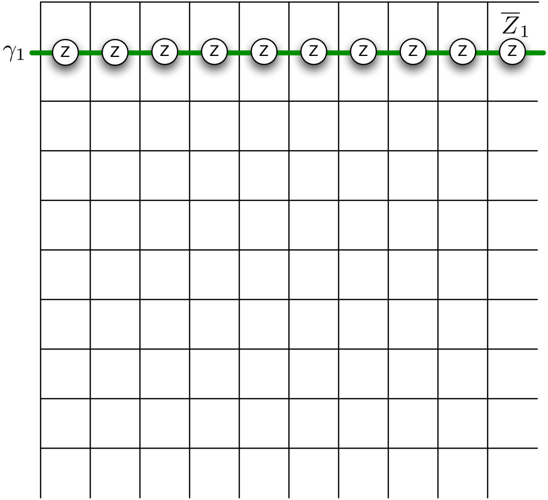

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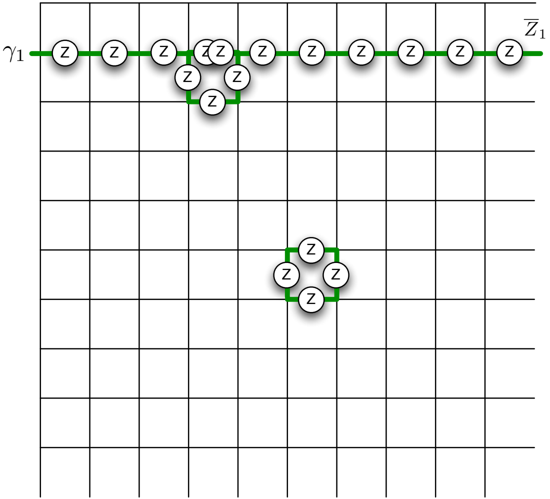

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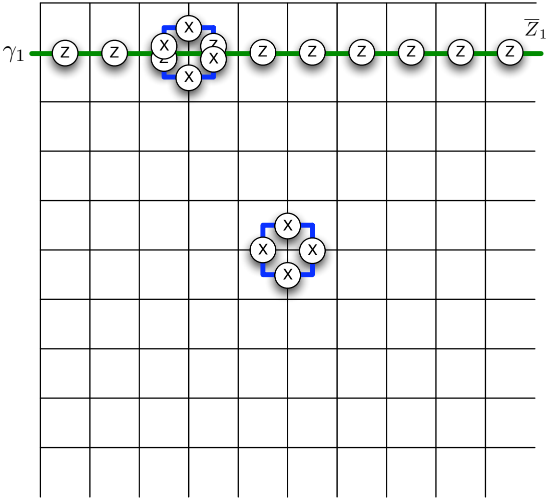

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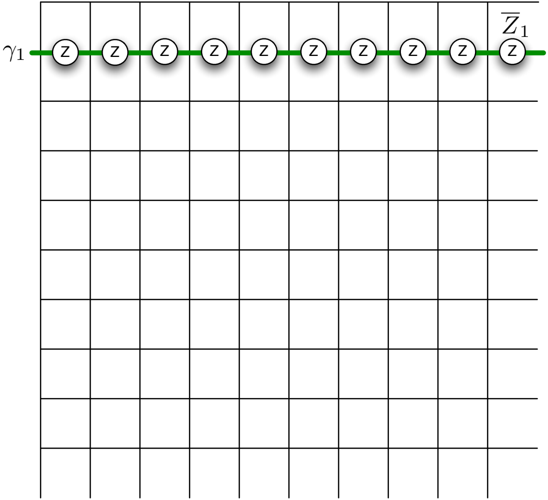

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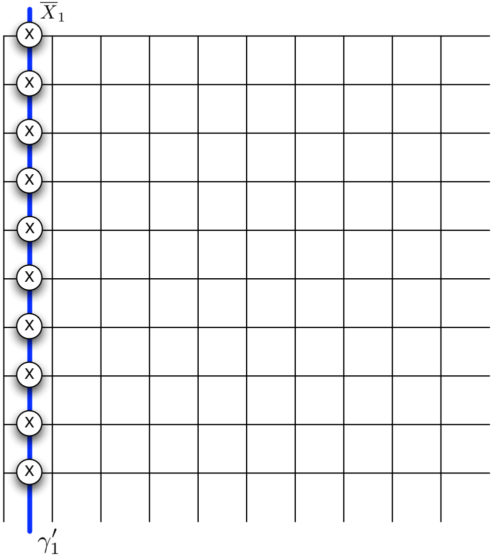

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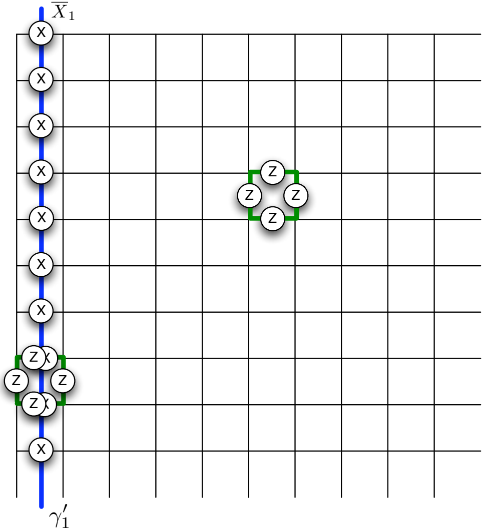

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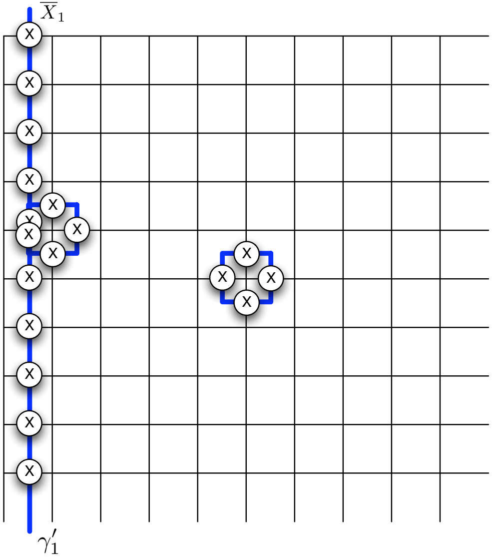

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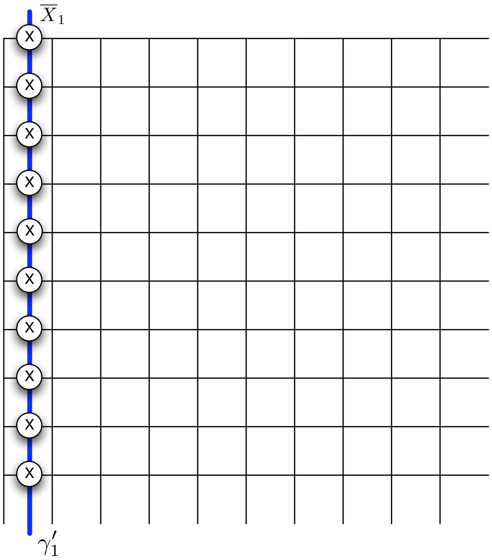

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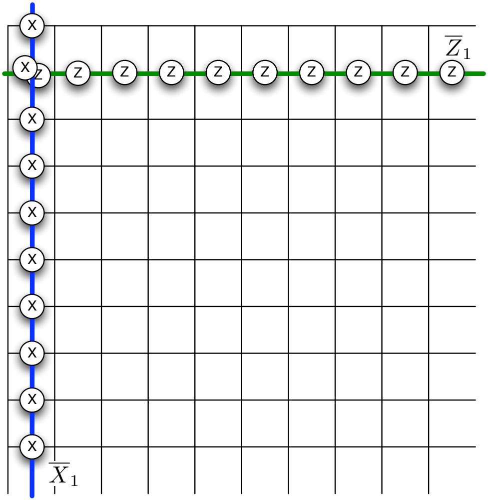

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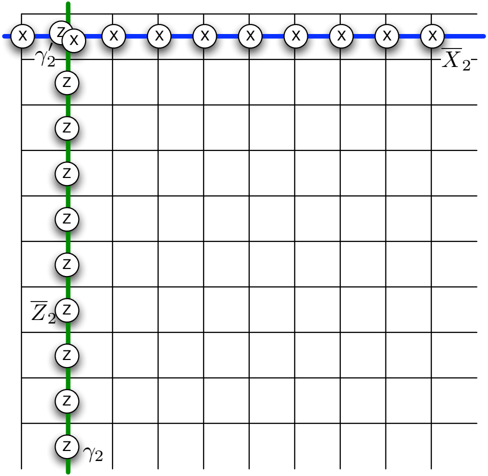

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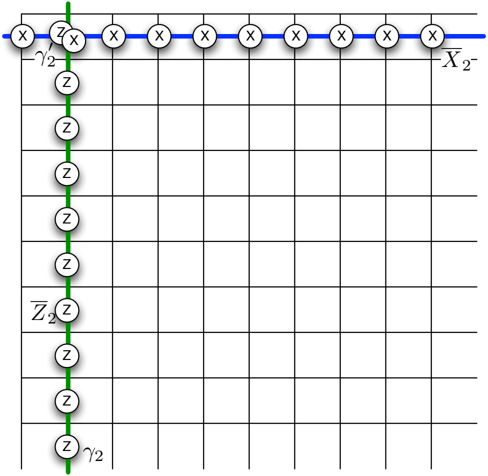

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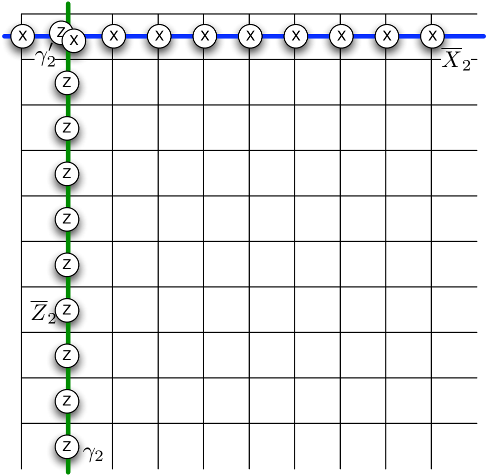

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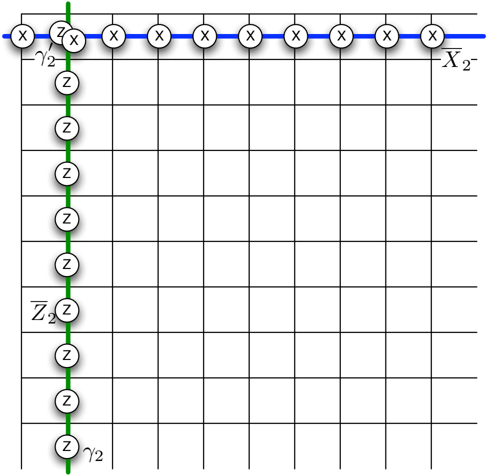

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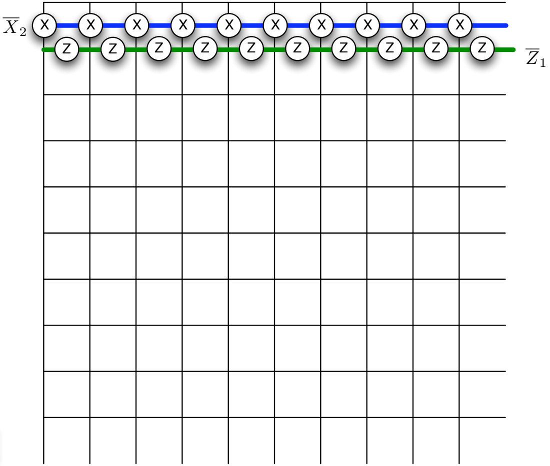

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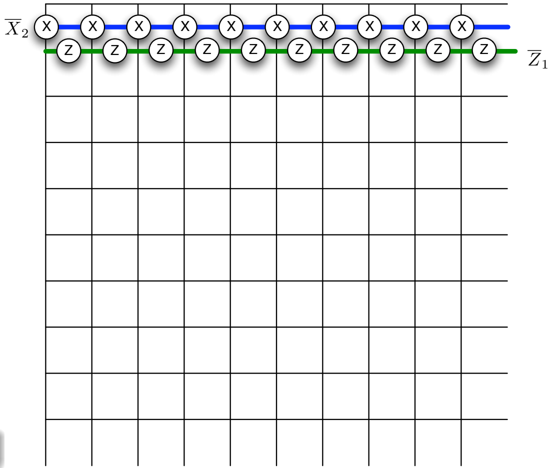
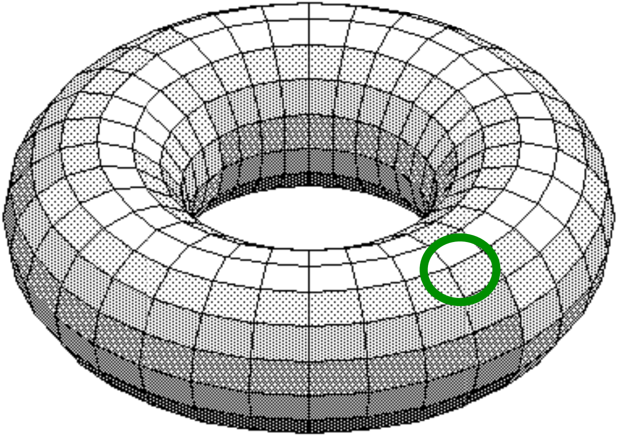


Two encoded qubits

Trivial cycles



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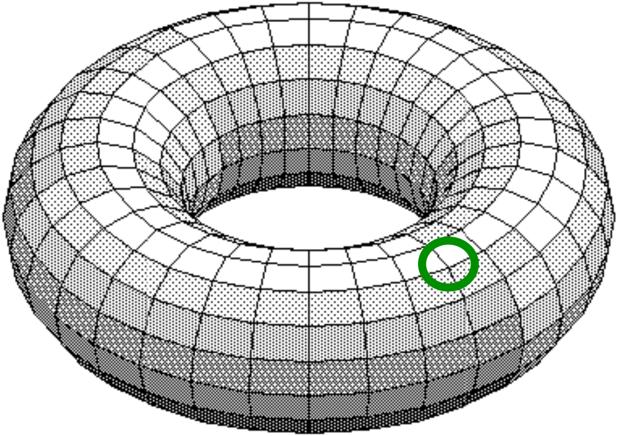

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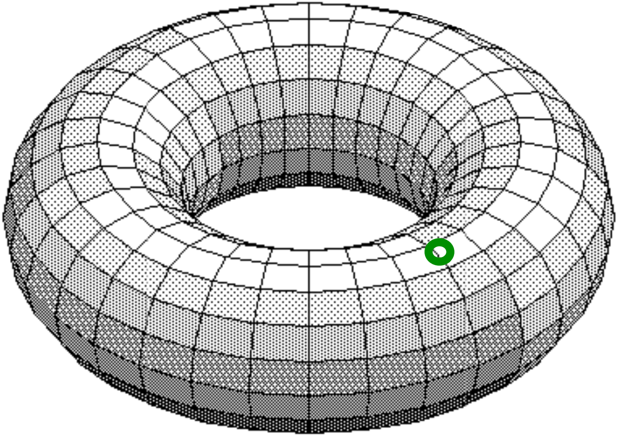

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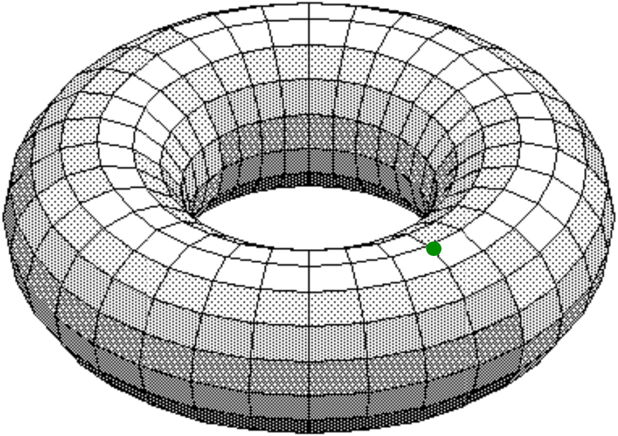

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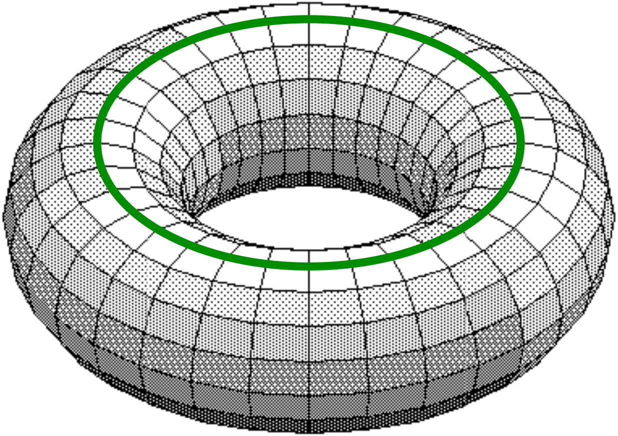

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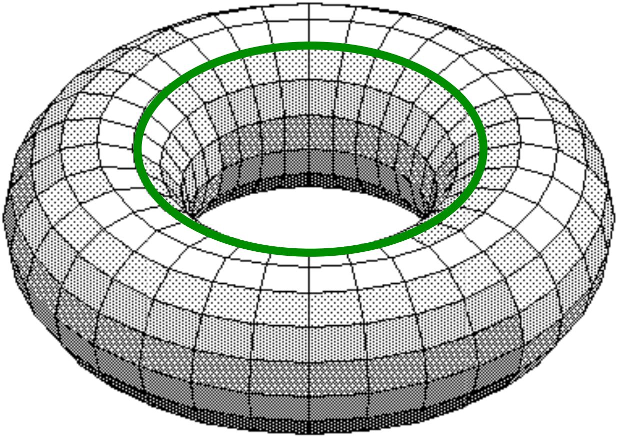

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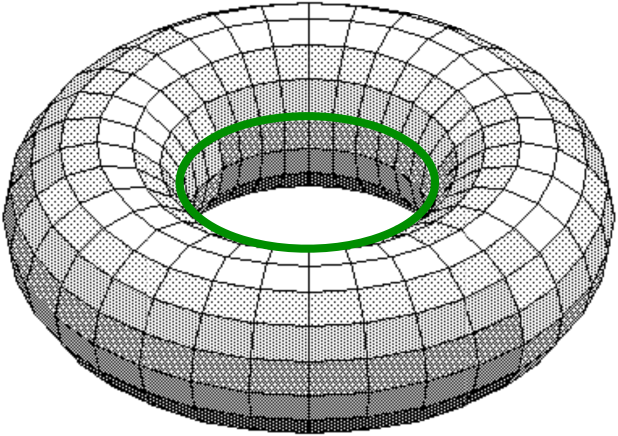

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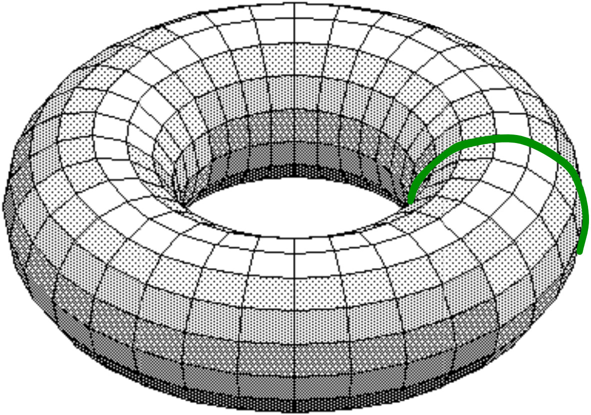
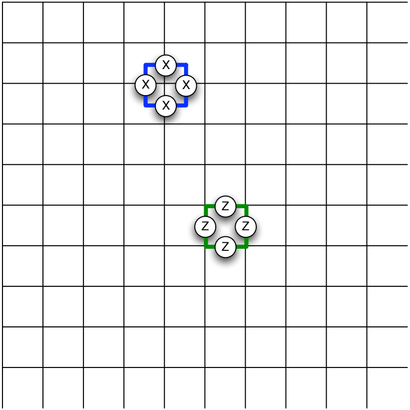


Trivial cycles and ground space

- $H = -(\sum_s A_s + \sum_p B_p)$
- The A_s et B_p are trivial cycles
- Trivial action on ground space
  $A_s|\psi\rangle = B_p|\psi\rangle = +1|\psi\rangle$
- A_s B_p generate all trivial loops.

Trivial loops act trivially on ground space



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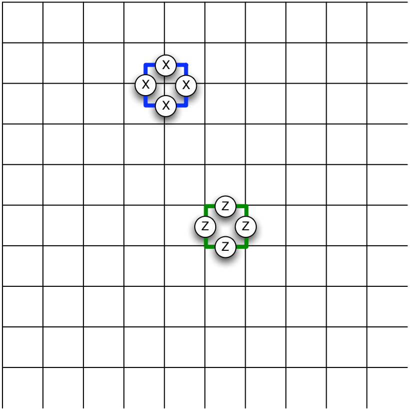

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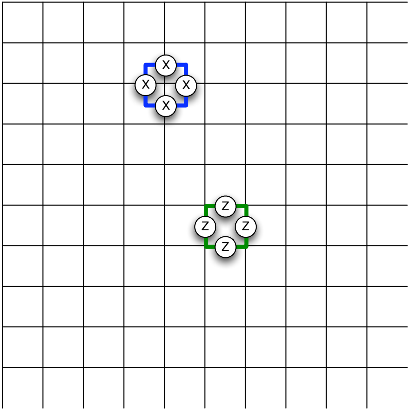

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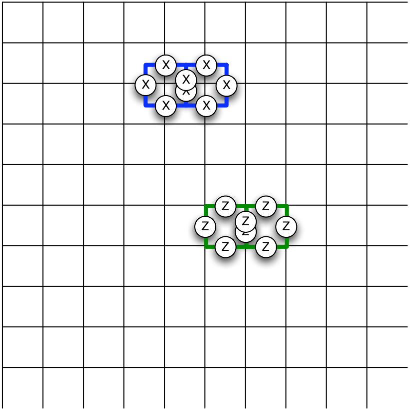

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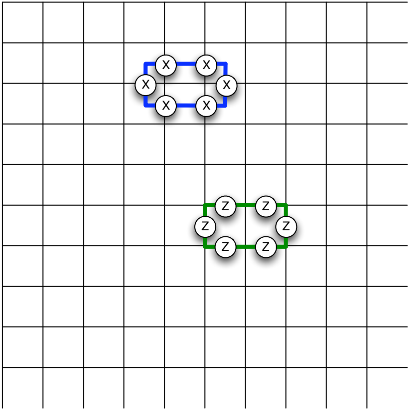

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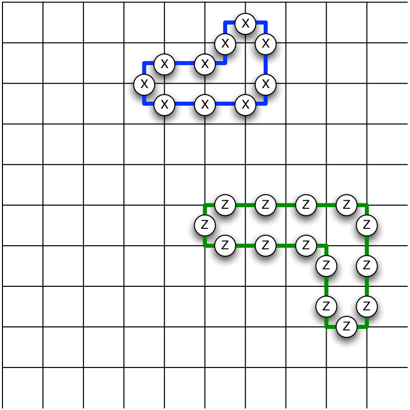

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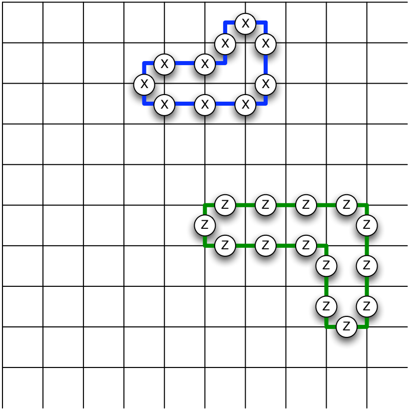
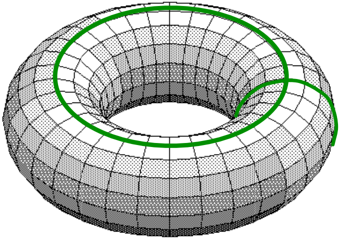
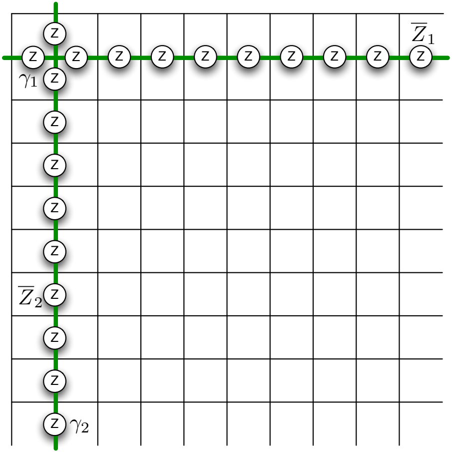


Non-trivial cycles

- γ_1 and γ_2 wrap around the torus: they are non-trivial cycles





Gauge choice

- $|\psi\rangle = B_{p'}|\psi\rangle$
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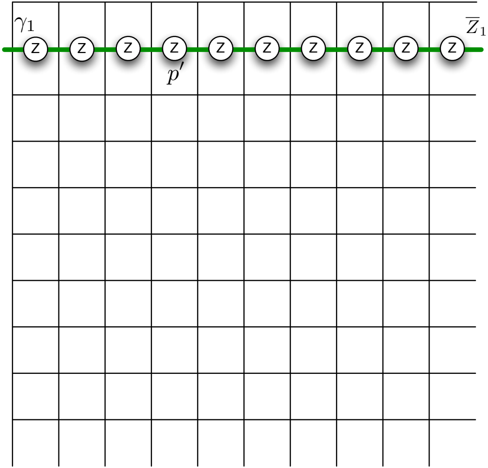

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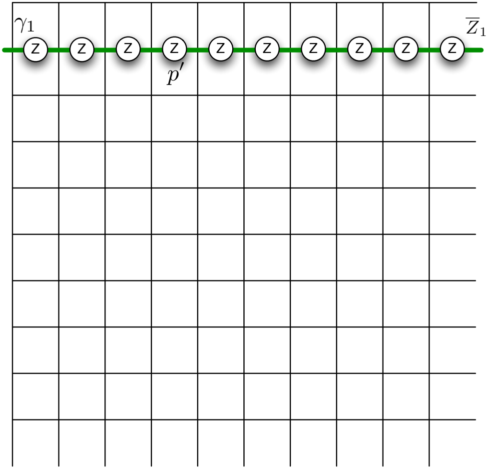

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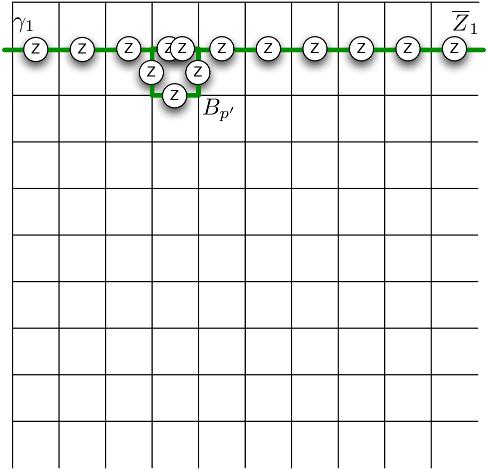

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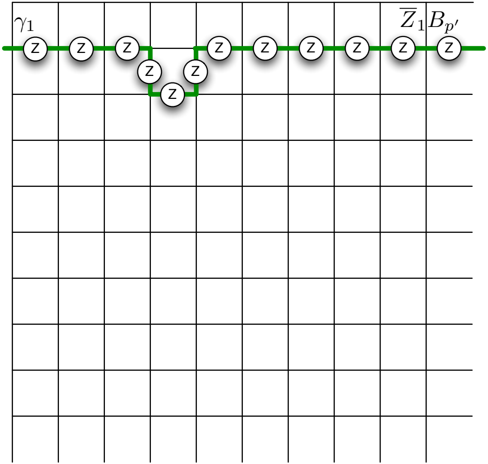

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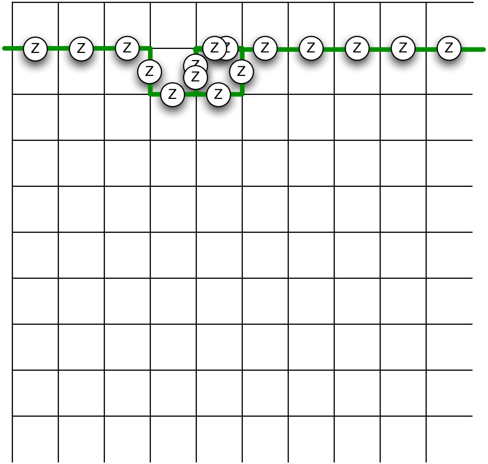

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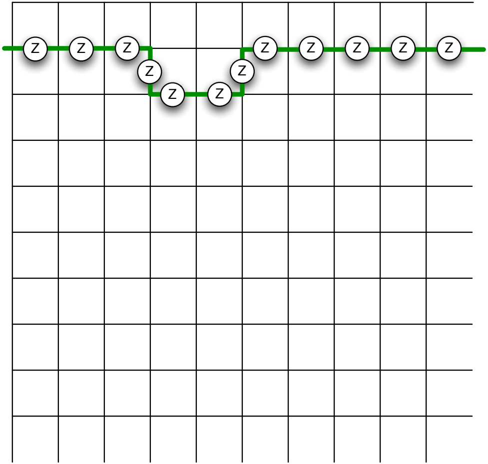

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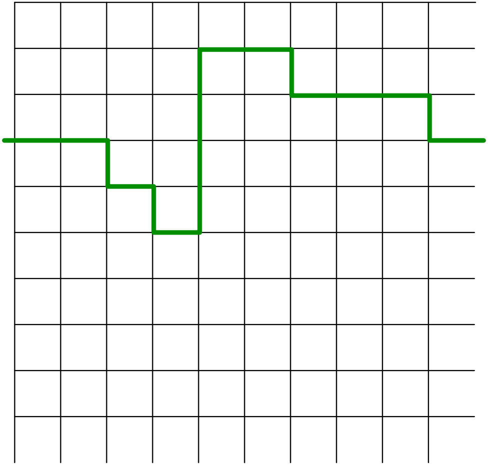
- $\overline{Z}_1|\psi\rangle = \overline{Z}_1 B_{p'}|\psi\rangle$
- $$\begin{aligned}\overline{Z}_1 &\equiv \overline{Z}_1 B_{p'} \\ &\equiv \overline{Z}_1 B_{p'} B_{p''} \\ &\equiv \overline{Z}_1 \prod_p B_p\end{aligned}$$



Hamiltonian - Topology

- One degree of freedom associated to each non-trivial cycle.
- Operator in same homological class act identically on ground space.
- We encode the quantum information is those degrees of freedom:
  - The information can only be modified by topologically non-trivial operators.
  - Robust when ($\ell \to \infty$)... ?

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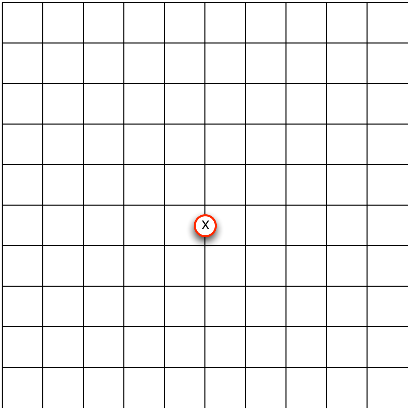
Particle creation

- Consider error $E = \sigma_x^i$.
- σ_x^i anti-commutes with adjacent plaquettes.
- $\sigma_x^i|\psi\rangle$ is a -1 eigenstate of B_p and $B_{p'}$
- Since $H = -(\sum_s A_s + \sum_p B_p)$, σ_x^i costs 2 energy units.
- This error has created a pair of magnetic particles.



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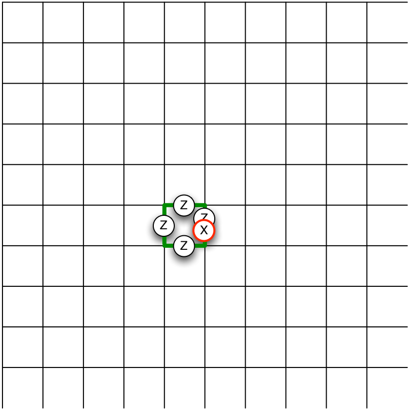

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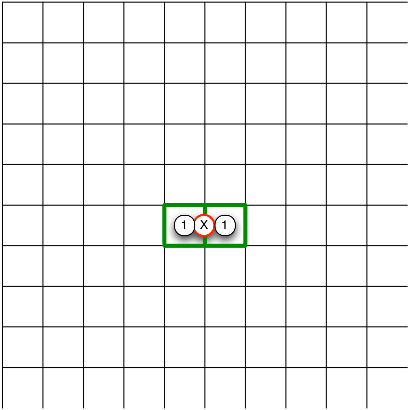

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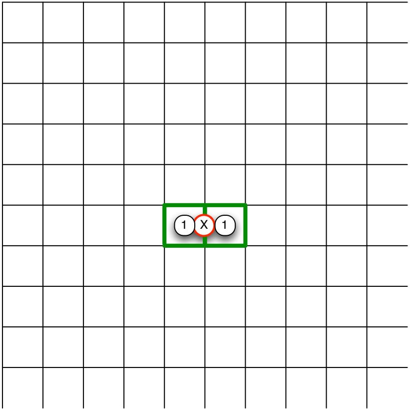

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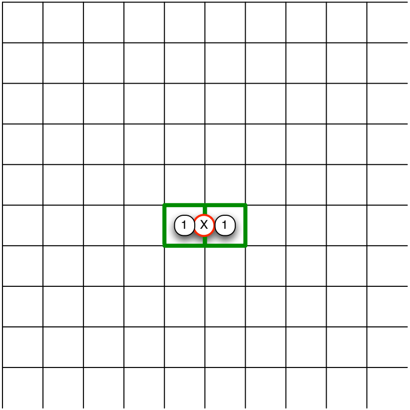

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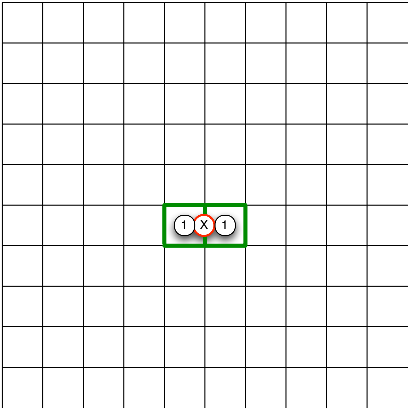
New error occurs on neighboring qubit:

- Restores the sign of the middle plaquette
- Flips the sign of the right plaquette

No net energy cost: particle has moved



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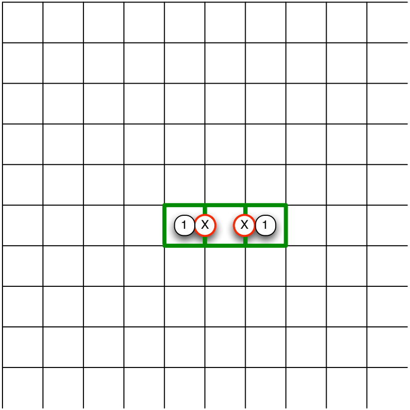

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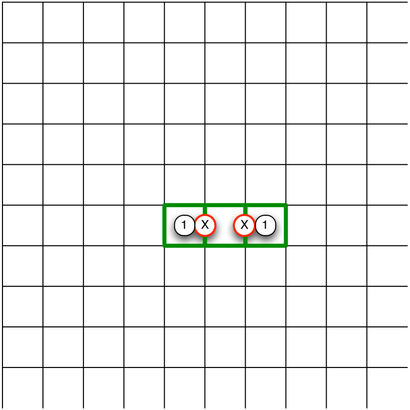

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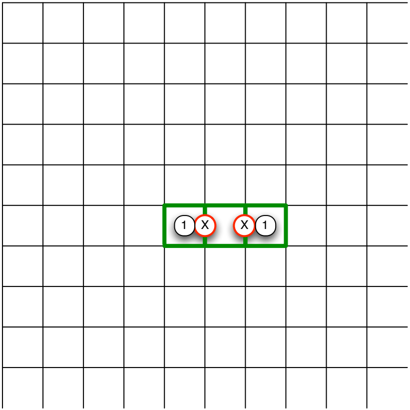
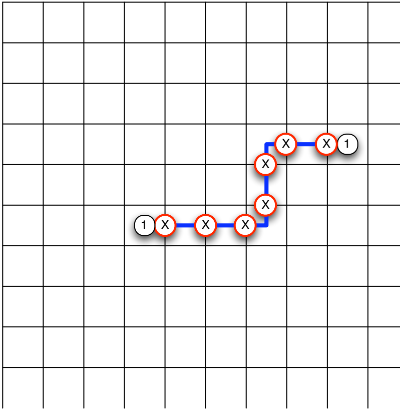


Error chains

- Error chains are attached to particles, each with given energy.
- Particles can move around at no energy cost.
- Error chains can be stretched freely.



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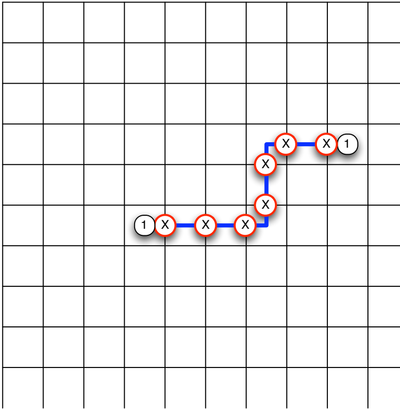

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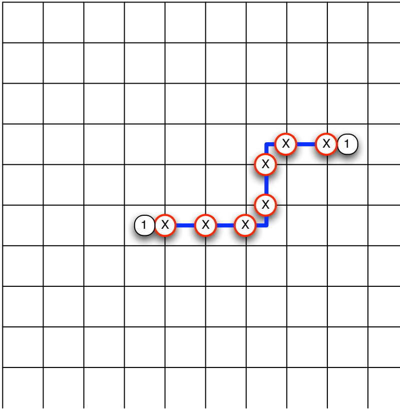
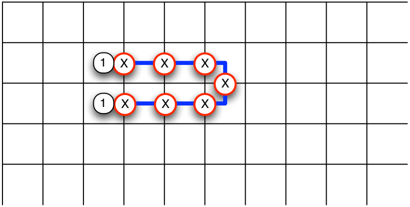
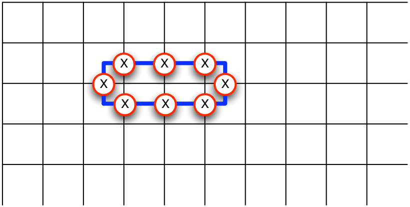


Particle annihilation

- An error can annihilate two particles
- The particle's worldline is left behind after fusion.
- Particle fusion can leave behind a worldline corresponding to a logical operation

Memory corruption





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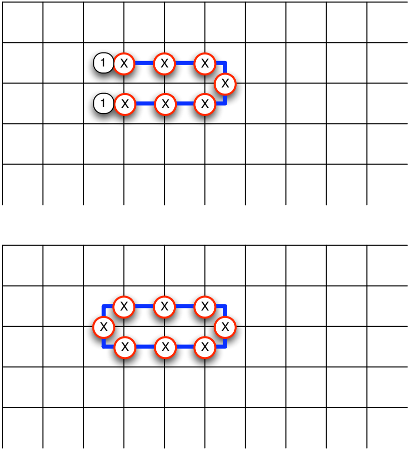

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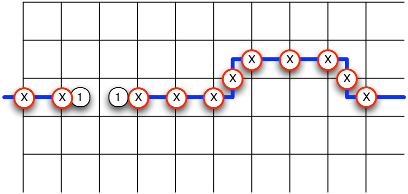

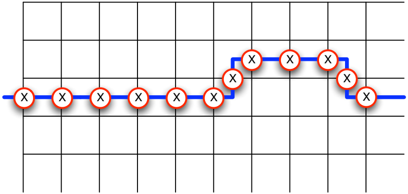

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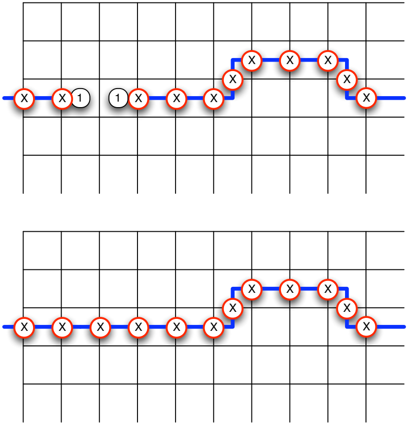
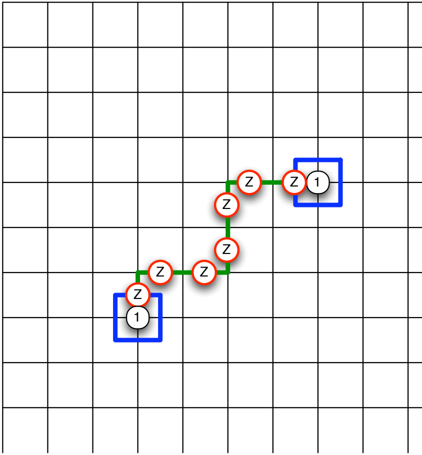


Electrical particles

- The same story holds for σ_z errors
- These will create electrical particles located at the lattice's vertices (plaquette of dual lattice).



Outline

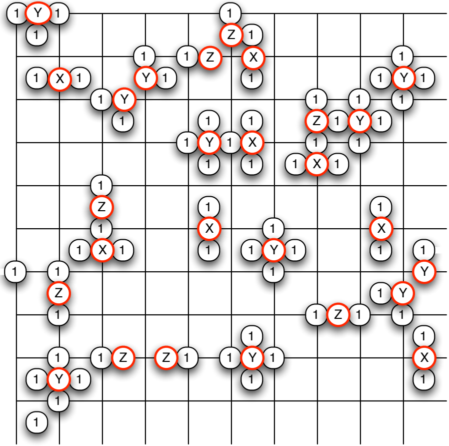
Error syndrome & decoding

- An error produces defects (error syndrome)
- Measure particle position, but not worldline.
- Many worldlines consistent with defects.
- Worldline with different homologies have different effect on ground space: MUST be distinguished.

Decoding

Infer worldline homology from particle location.

15 % Noise rate



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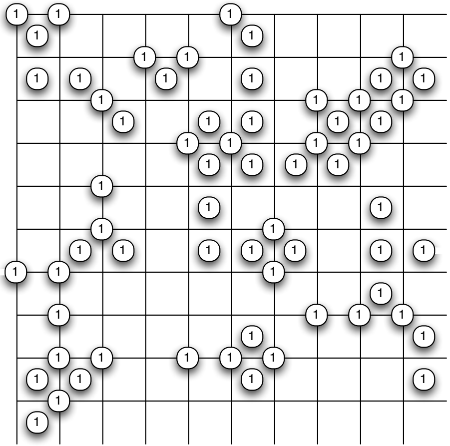

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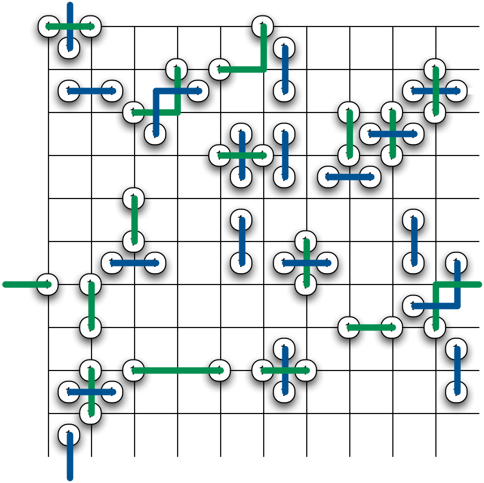

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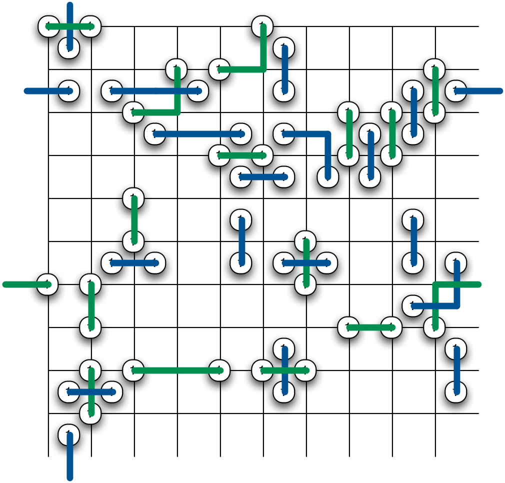

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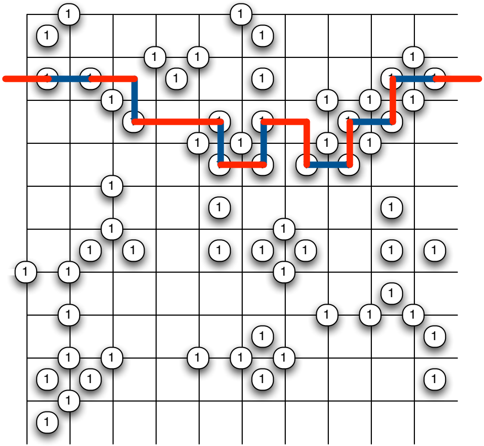

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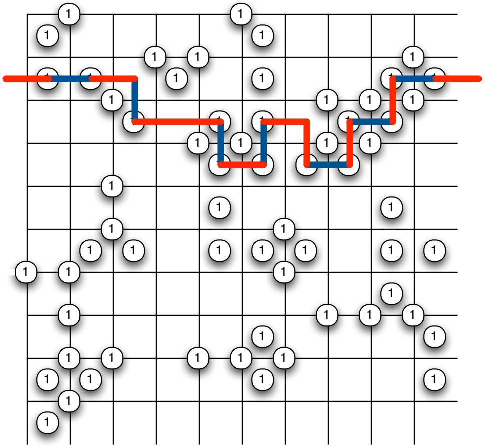

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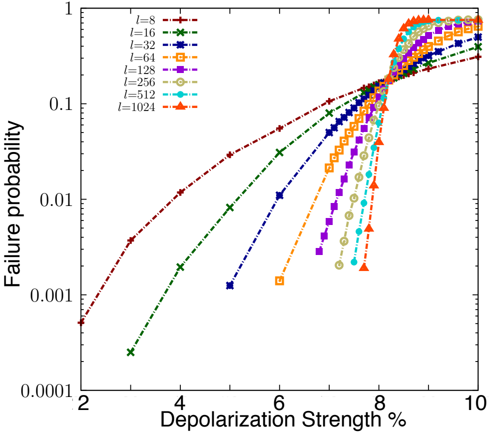

- Threshold noise rate $\approx 8.2\%$.

Known self-correcting systems

- 2D toric code has point-like electric and magnetic excitations, ends of error strings.
- 3D toric code has point-like electric excitations and string-like magnetic excitations, boundaries of error membranes.
  - Z type errors are confined due to string tension.
- 4D toric code has sting-like electric and magnetic excitations, boundaries of error membranes.
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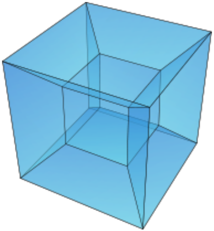

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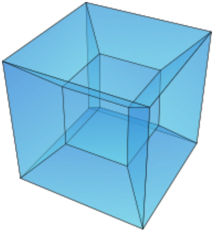

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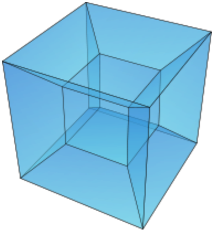

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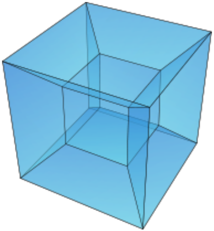

Confinement in lower dimensions?

Main idea

Find a local hamiltonian with topological order but with confined excitations.

- Add an attractive potential between topological excitations.
  - Can be realized by coupling to bosonic field (phonons).
- Exists a finite temperature confined phase?
- Destroys topological order?
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- etc.

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Outline

Definitions

- Λ is a 2D lattice.
- Each vertex occupied by d-level quantum particle.
- Hamiltonian $H = -\sum_{X \subset \Lambda} P_X$ with
  - $P_X = 0$ if radius$(X) \geq w$.
  - $[P_X, P_Y] = 0$.
  - P_X are projectors (optional).
- Code $\mathcal{C} = \{\psi : P_X|\psi\rangle = |\psi\rangle\}$
  = ground space of H
  = image of code projector $\Pi = \prod_X P_X$
- With proper coarse graining, we can assume that
  - Λ is a regular square lattice.
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  - P_X are projectors (optional).
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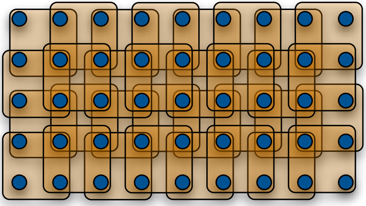

Well known examples

- Kitaev's toric code
- Bombin's topological color codes
- Levin & Wen's string-net models
- Turaev-Viro models
- Kitaev's quantum double models
- Most known models with topological quantum order

Remark

The first two example are simple because they are stabilizer codes. Most things I will say are trivial to prove in this case.

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Subsystem codes do not belong to this family.

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Standard definitions

Correctable region

A region $M \subset \Lambda$ is *correctable* if there exists a recovery operation $\mathcal{R}$ such that $\mathcal{R}(\mathrm{Tr}_M \rho) = \rho$ for all code states ρ.
M correctable $\Leftrightarrow$ No order parameter on M $\Leftrightarrow$ $\Pi O_M \Pi \propto \Pi$.

Minimum distance

The minimum distance d is the size of the smallest non-correctable region.

Logical operator

Operator L such that $L|\psi\rangle$ is a code state for any code state $|\psi\rangle$.

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Outline

Statement of the lemma

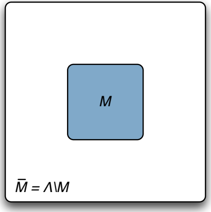
Holographic disentangling lemma (Bravyi, DP, Terhal)

Let $M \subset \Lambda$ be a correctable region and suppose that its boundary ∂M is also correctable. Then, there exists a unitary operator $U_{\partial M}$ acting only on the boundary of M such that, for any code state $|\psi\rangle$,

$$U_{\partial M}|\psi\rangle = |\phi_M\rangle \otimes |\psi'_{\overline{M}}\rangle$$

for some *fixed* state $|\phi_M\rangle$ on M.

With pictures



- Let M be correctable.
- Assume ∂M is correctable.
- Let $M = A \cup B$, $\overline{M} = C \cup D$, and $\partial M = B \cup C$.

- There exists a unitary transformation $U_{\partial M}$ such that, for any $|\psi\rangle \in C$

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For a trivial code $\mathrm{Tr}\Pi = 1$, every region is correctable, so we recover the area law $S(M) \leq |\partial M|$ for commuting Hamiltonians of Wolf, Verstraete, Hastings, and Cirac.

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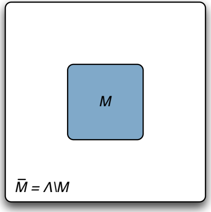

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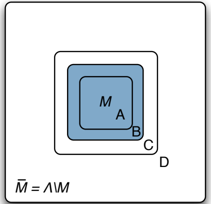

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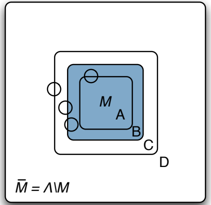

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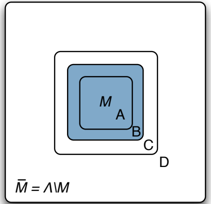

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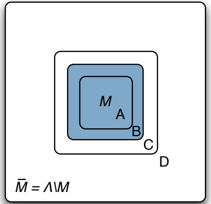

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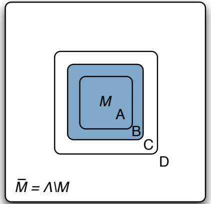

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Statement of the result

String-like logical operators (Haah, Preskill)

There exists a non-trivial logical operator supported on a string-like region.

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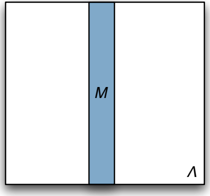

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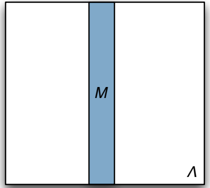

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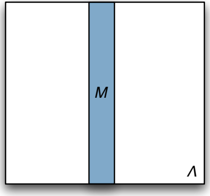

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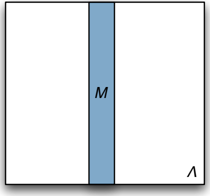

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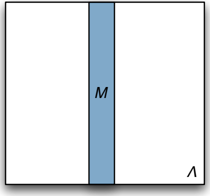

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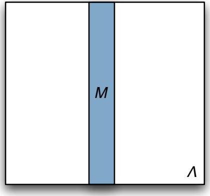

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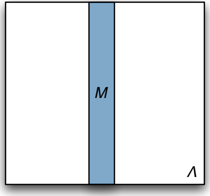

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Thermal instability

- Local commuting projector codes have string-like logical operators.
- If this logical operator is a sequence of local unitary operators, system is thermally unstable.
  - We can sequentially apply the transformation to create, move, and fuse a point-like excitation.
- What happens in more general cases?

Main result (Landon-Cardinal & DP)

The minimum set of conditions required to prove spectral stability imply the existence of a sequence of local maps that corrupt the system at an energy cost bounded by a constant.

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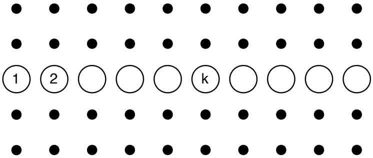
Noise model



1. Apply random unitary on sites 1 & 2.
2. Measure P_{12}
   - If $P_{12} = 0$ go to 1.
3. Apply random unitary on site 3.
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- Only a constant amount of energy at any given time.
- No need to backtrack.
- Number of steps $\propto$ lattice linear size.
- If successful, final state is corrupted. (not trivial)

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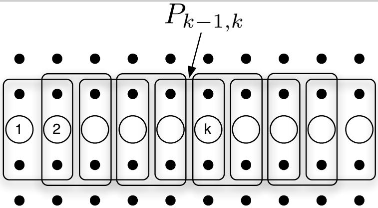

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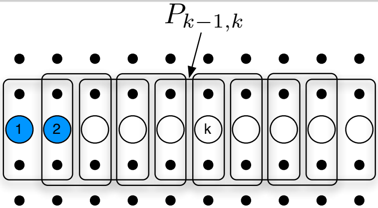

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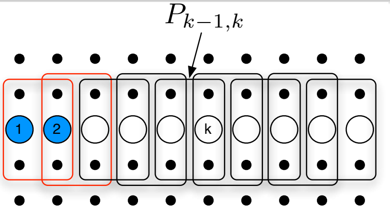

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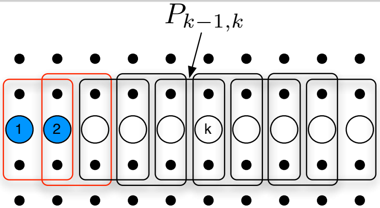

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- No need to backtrack.
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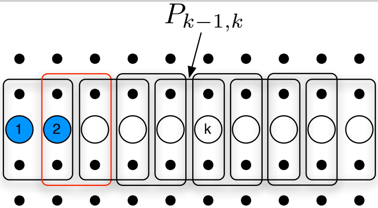
Noise model



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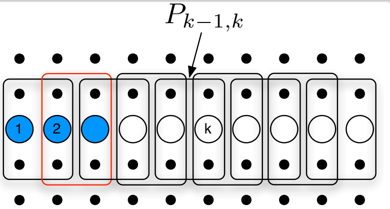

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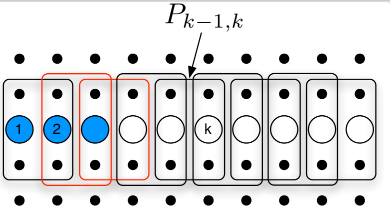

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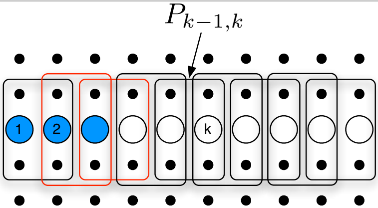

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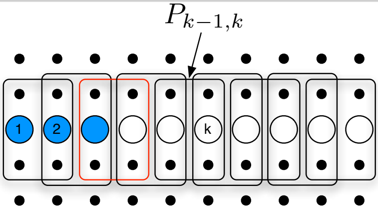

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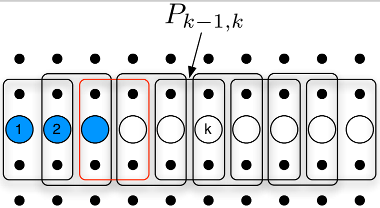

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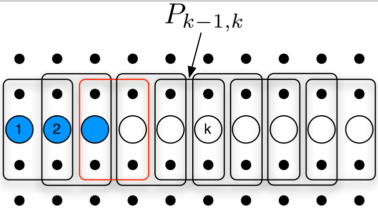

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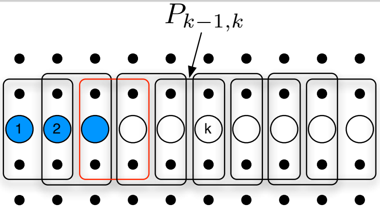

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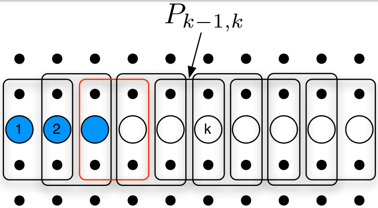

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Outline

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  - Stable spectrum $\Rightarrow$ topological order
  - Finite temperature phase $\Rightarrow$ energy barrier
- Conflict in two spacial dimensions.
- Many connections between coding theory and many-body physics
  - Decoding problem $\Leftrightarrow$ Renormalization group methods
  - Fault-tolerant threshold $\Leftrightarrow$ ordered-disordered transition
  - Error correction $\Leftrightarrow$ Topological order (no local order parameter)
  - Holographic disentangling lemma $\Leftrightarrow$ Area law

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