Recent new progress on variational approach for strongly correlated t-J model

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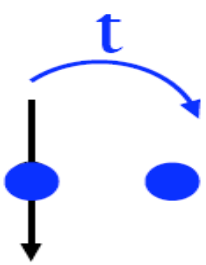
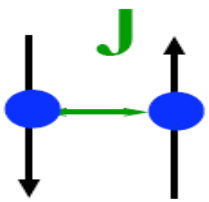
The minimal model
-- extended t-J Hamiltonian

$$-\sum_{i,j,\sigma} t_{ij}\left(\tilde{c}^{\dagger}_{i,\sigma}\tilde{c}_{j,\sigma} + h.c.\right) + J\sum_{<i,j>} \vec{S}_i \bullet \vec{S}_j$$

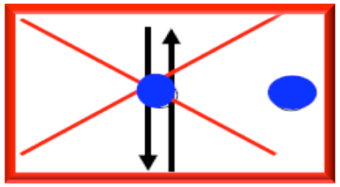


t_{ij} = n.n.(t), 2nd n.n.(t'), 3rd n.n.(t'') hopping

J = n.n. AF spin-spin interaction



Strong constraint -- no two electrons on the same site



Use a variational Monte Carlo approach to satisfy the constraint

$$\left|\Psi\right\rangle = \hat{P}_J \prod_{R_i}\left(1 - n_{R_i\uparrow} n_{R_i\downarrow}\right)\left|\Psi_0\right\rangle \qquad \mathrm{P_d} = \prod_i (1 - n_{i\uparrow} n_{i\downarrow})$$

$\hat{P}_J$ **-- hole-hole repulsive Jastrow factor!**

Outline

Use variational approach

Part I:
To study strongly correlated superconducting state with spatial inhomogeneity : a theory for stripes in high temperature superconductors

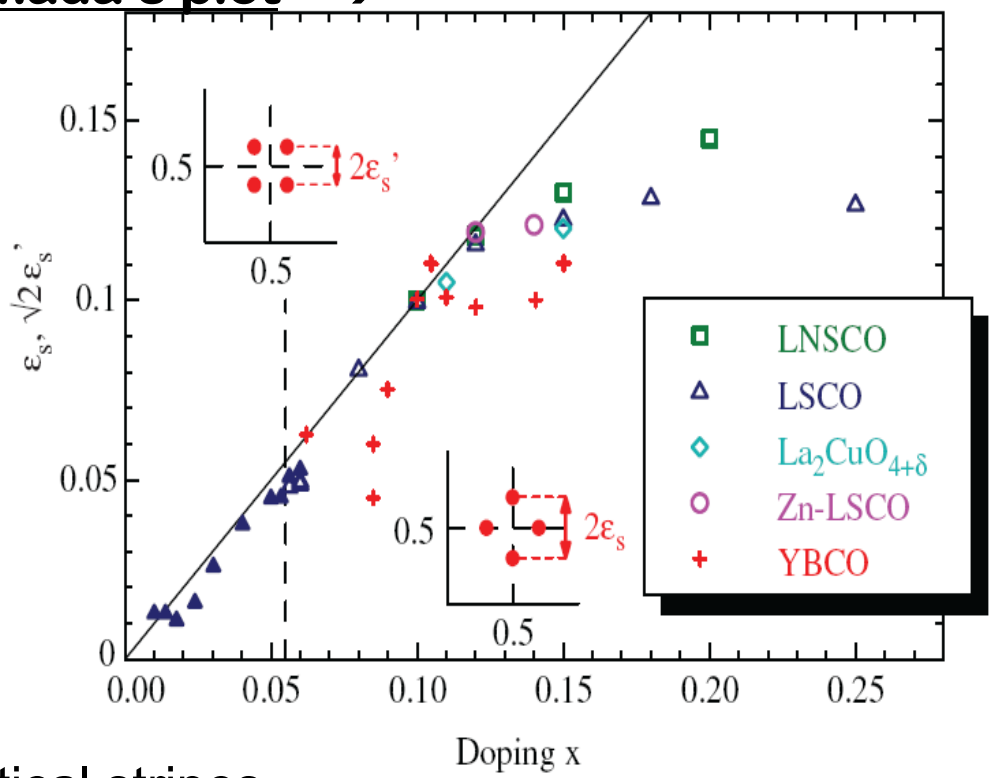
Part II:

To study phase fluctuation of the strongly correlated superconducting state

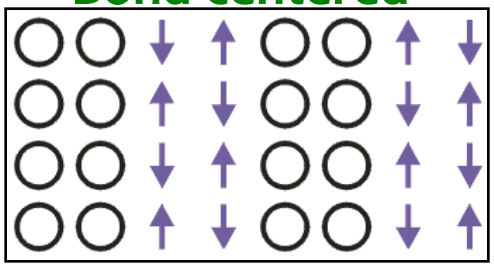
Neutron scattering – probing spin ordering

Magnetic period vs Doping → Half-doped Stripe !

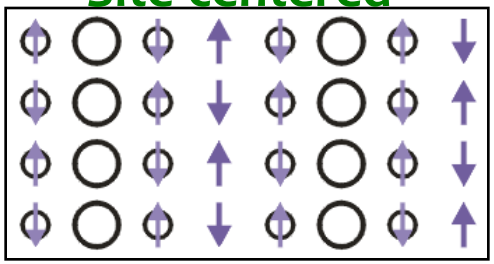
Yamada's plot →



Bond-centered



Site-centered



Example of x=1/8 doping,
magnetic modulation period is 1/x
Charge modulation is 1/2x

For vertical stripes

$$x = \varepsilon_S = 1/a_S$$

Yamada, Fujita,
Tranquada, …

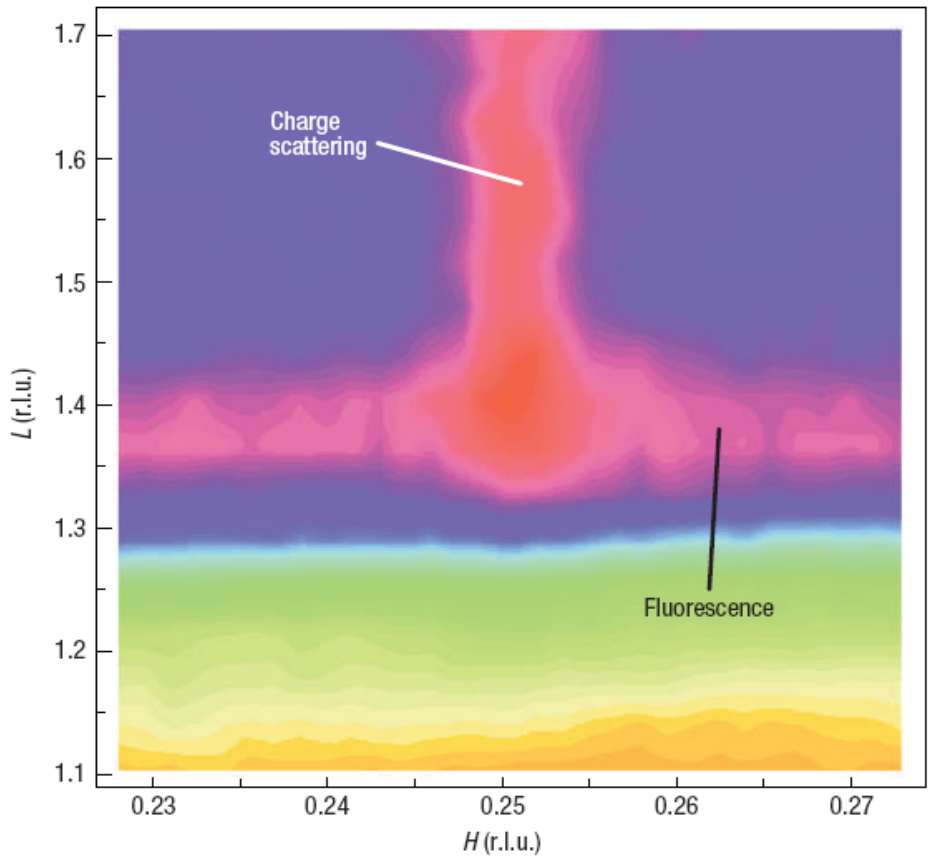
Vojta, Adv. in Phys. '09

Soft X-ray scattering – probing charge ordering

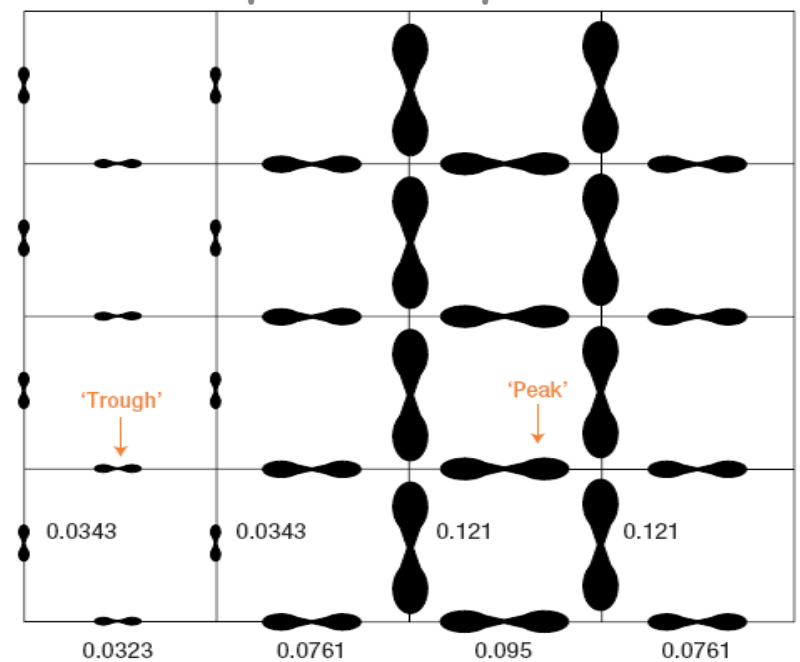
$La_{1.875}Ba_{0.125}CuO_4$

$a_S = 2a_C$

Charge correlation in (H,0,L) plane

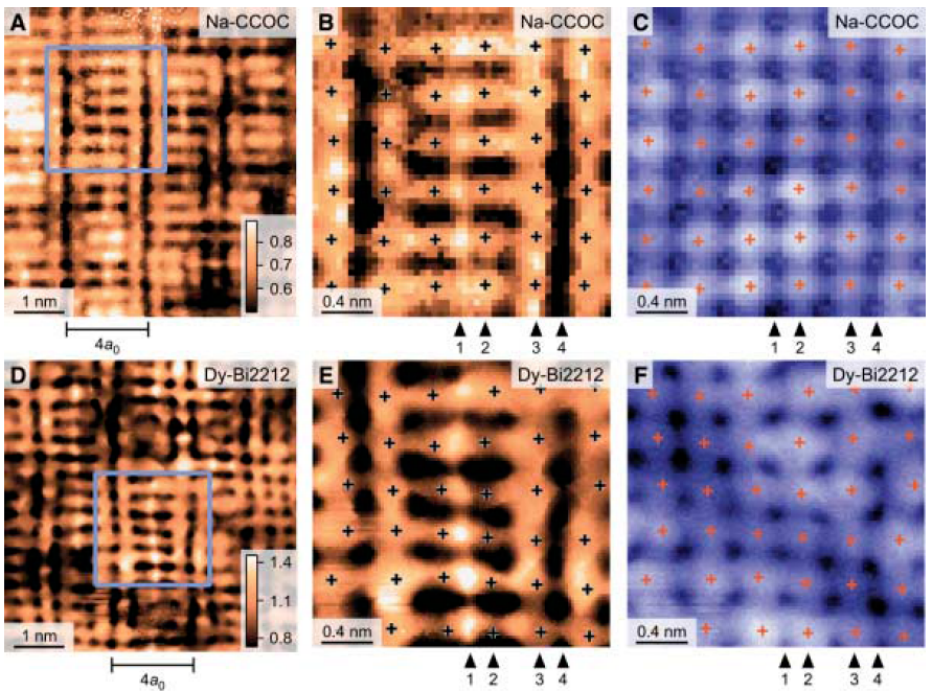


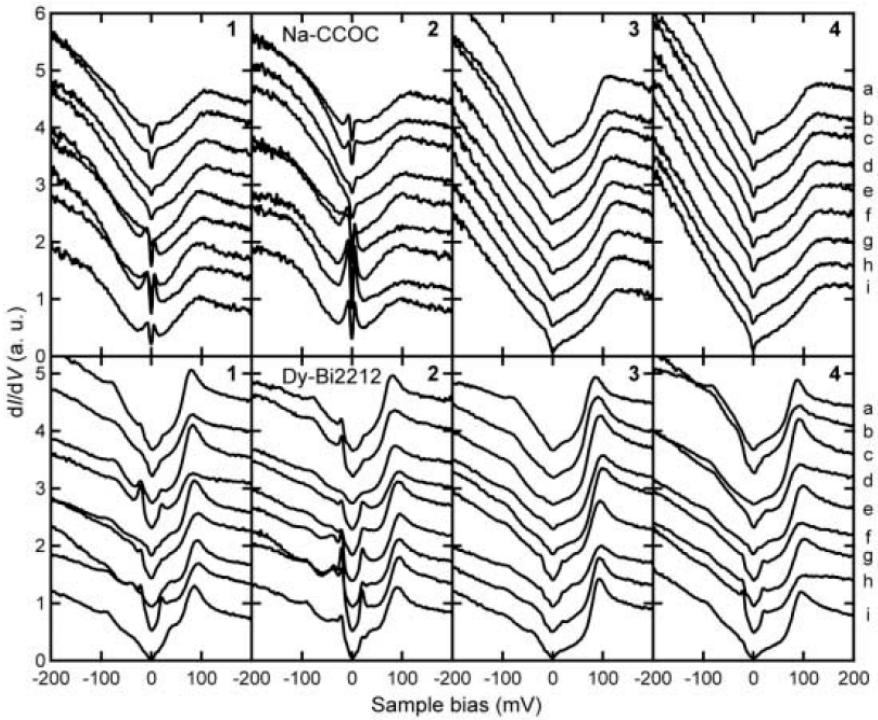
Stripe orbital pattern



Modulation period: $4a_0$

Abbamonte, *et al.*, Nature Physics '05





Evidence for bond-centered electronic cluster glass state with unidirectional $4a_0$ domains!

V-shape LDOS !

Kohsaka, *et al.*, Science '07

Three main views about the formation mechanism of stripes:

1. Usual CDW or SDW needs a nesting wave vector and strong lattice coupling → a competing interaction? nesting vector?
– Moskiewicz, '99

2. Hubbard or t-J model favors phase separation and long-range Coulomb interaction frustrates the phase separation and leads to stripes → stability? why half-doped?
– Emery, Kivelson, Lin, Fradkin, *et. al.*, and C. Di Castro, *et al.*

3. Competition between kinetic and exchange energies in Hubbard or t-J model
– HF --- Zaanan, Poilblanc and Rice….
– DMRG --- White and Scalapino (t' or 2nd n.n. hopping suppresses stripes)
– VMC --- Kobayashi and Yamada; Miyazaki *et al.*; Himeda, Kato and Ogata (t' stabilizes stripes)…

To treat these competing states quantitatively and reliably ---- variational approach!

For a translational invariant state, it is straightforward to construct a variational wave function for a projected d-wave state or resonanting valence bond state (by P. W. Anderson)

$$|RVB\rangle = \mathrm{P_d}\left[\prod_k (u_k + v_k C^+_{k,\uparrow} C^+_{-k,\downarrow})\right]|0\rangle = \mathrm{P_d}|BCS\rangle$$

The Gutzwiller operator $\mathrm{P_d}$ enforces no doubly occupied sites for hole-doped systems

$$\mathrm{P_d} = \prod_i (1 - n_{i\uparrow} n_{i\downarrow})$$

$$P_{N_e}|RVB\rangle = \mathrm{P_d}\left(\sum_k \frac{v_k}{u_k} C^+_{k,\uparrow} C^+_{-k,\downarrow}\right)^{N_e/2}|0\rangle = \mathrm{P_d}\left(\sum_{i,j} a_{i,j} C^+_{i,\uparrow} C^+_{j,\downarrow}\right)^{N_e/2}|0\rangle$$

$$v_k / u_k = \frac{E_k - \xi_k}{\Delta_k}, \Delta_k = \Delta(\cos k_x - \cos k_y)$$

$$\xi_k = -2(\cos k_x + \cos k_y) - 4t'_v \cos k_x \cos k_y - 2t''_v(\cos 2k_x + \cos 2k_y) - \mu_v ,$$

$$E_k = \sqrt{\xi_k^2 + \Delta_k^2}$$

How to include the inhomogeneity into the wave function?

1. Solve the BdG equations and obtain the eigenvectors,

$$\sum_j \begin{pmatrix} H_{ij\uparrow} & F_{ij} \\ F^*_{ji} & -H_{ji\downarrow} \end{pmatrix} \begin{pmatrix} u^n_j \\ v^n_j \end{pmatrix} = E_n \begin{pmatrix} u^n_i \\ v^n_i \end{pmatrix}$$

2. Make Bogoliubov transformation,

$$\begin{pmatrix} \gamma_n \\ \bar{\gamma}_n \end{pmatrix} = \begin{pmatrix} u^n_i & v^n_i \\ \bar{u}^n_i & \bar{v}^n_i \end{pmatrix} \begin{pmatrix} c_{i\uparrow} \\ c^\dagger_{i\downarrow} \end{pmatrix}, \qquad \begin{matrix} E_n > 0 \\ \bar{E}_n < 0 \end{matrix}$$

3. Construct the trial wave function,

$$\hat{P}_h \hat{P}_{N_e} |\Phi_0\rangle = \hat{P}_h \hat{P}_{N_e} \prod_n \gamma_n \bar{\gamma}^\dagger_n |0\rangle \propto \hat{P}_h \left(\sum_{i,j} (\hat{U}^{-1}\hat{V})_{ij} c^\dagger_{i\uparrow} c^\dagger_{j\downarrow} \right)^{N_e/2} |0\rangle$$

Himeda *et al.*, PRL '02

Matrix size: 2N x 2N reduced to N x N

4. To optimize energy, change to a new set of parameters, back to steps 1,2,3..

$$\hat{P}_h \hat{P}_{N_e} \prod_n \gamma_n \bar{\gamma}_n^\dagger |0\rangle \propto \hat{P}_h \left(\sum_{i,j} \left(\hat{U}^{-1} \hat{V} \right)_{ij} c_{i\uparrow}^\dagger c_{j\downarrow}^\dagger \right)^{N_e/2} |0\rangle$$

Himeda *et al.*, PRL '02

There is a problem with this construction as d-wave pairing has a node, hence there is a possibility to hit a singular value.

A new way to express the wave function…

Yokoyama and Shiba, JPSJ 1988

Particle-hole (P-H) transformation

$$c_{i\uparrow} \Rightarrow f_i$$

$$c_{i\downarrow} \Rightarrow d_i^\dagger$$

$$\hat{P}_h \hat{P}_{N_e} \prod_n \gamma_n \bar{\gamma}_n^\dagger |0\rangle \propto \hat{P}_h \hat{P}_{N_e} \prod_n \left(\sum_i u_i^n c_i^\dagger + v_i^n d_i^\dagger \right) |\tilde{0}\rangle$$

Construct stripe $|\Psi_0\rangle$ for hole-doped states

Mean-field Hamiltonian

$$H_{ij\sigma} = -\sum_{\alpha=n,nn,nnn} t_{ij}\delta_{i+\alpha,j} + \left(\rho_i + \sigma m_i (-1)^{R_i^x + R_i^y} - \mu\right)\delta_{ij}$$

$$H_{MF} = \begin{pmatrix} c_{i\uparrow}^{\dagger} & c_{i\downarrow} \end{pmatrix} \begin{pmatrix} H_{ij\uparrow} & \Delta_{ij} \\ \Delta_{ji} & -H_{ji\downarrow} \end{pmatrix} \begin{pmatrix} c_{j\uparrow} \\ c_{j\downarrow}^{\dagger} \end{pmatrix}$$

3 new parameters,

Pair period

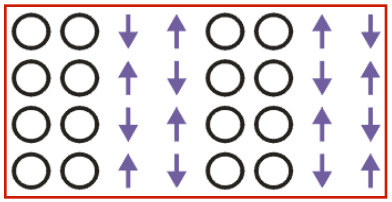
Charge period

$$\boxed{a_C, a_P, a_s}$$

Spin period

AF-RVB stripe [In-Phase-Δ and Anti-Phase-m]

Bond-centered



Charge: $\rho_i = \rho_v \cos\left[\vec{Q}_C\left(\vec{R}_i - \vec{R}_0\right)\right]$

Spin: $m_i = m_v \sin\left[\vec{Q}_S\left(\vec{R}_i - \vec{R}_0\right)\right]$

$$\vec{Q}_\alpha = \left(Q_x, Q_y\right) = \left(0, \frac{2\pi}{a_\alpha}\right), \quad \alpha = C, P, S$$

$$\vec{R}_0 = \left(0, \frac{1}{2}\right), \quad \vec{R}_i = \left(R_i^x, R_i^y\right)$$

Pair:

$$\Delta_{i,i+x} = \Delta_v^M \cos\left[\vec{Q}_P\left(\vec{R}_i - \vec{R}_0\right)\right] - \Delta_v^C$$

$$\Delta_{i,i+y} = -\Delta_v^M \cos\left[\vec{Q}_P\left(\vec{R}_i - \vec{R}_0\right) + Q_y\right] + \Delta_v^C$$

The stripe states will involve charge density and spin density modulation, it will also involve pair field. What is the relation between these three quantities? Their relative periods?

Gutzwiller's approximation in t-J model

$$H_{t-J} = -t \sum_{<i,j>,\sigma} \left(c_{i\sigma}^{\dagger} c_{j\sigma} + h.c. \right) + J \sum_{<i,j>} S_i \bullet S_j$$

$$\left\langle S_i \bullet S_j \right\rangle = g_s(i) g_s(j) \left\langle S_i \bullet S_j \right\rangle_0$$

$$\left\langle c_{i\sigma}^{\dagger} c_{j\sigma} \right\rangle = g_{t\sigma}(i) g_{t\sigma}(j) \left\langle c_{i\sigma}^{\dagger} c_{j\sigma} \right\rangle_0$$

Mean-field decoupling…

$$\left\langle H_{t-J} \right\rangle = -t \sum_{<i,j>,\sigma} g_{t\sigma}(i) g_{t\sigma}(j) \left(\chi_{ij\sigma} + c.c. \right) - J \sum_{<i,j>} g_s(i) g_s(j) \left(\frac{3}{8} \left(\chi_{ij} \chi_{ij}^{*} + \Delta_{ij} \Delta_{ij}^{*} \right) - m_i m_j \right)$$

$$g_{t\sigma}(i) = \left(1 - n_{i\bar{\sigma}}\right) \sqrt{\frac{n_i \left(1 - n_i\right)}{\left(1 - n_{i\uparrow}\right)\left(1 - n_{i\downarrow}\right)\left(n_i - 2 n_{i\uparrow} n_{i\downarrow}\right)}}$$

$$g_s(i) = \frac{n_i}{n_i - 2 n_{i\uparrow} n_{i\downarrow}}$$

$$\chi_{ij\sigma} = \left\langle c_{i\sigma}^{\dagger} c_{j\sigma} \right\rangle_0 \qquad \Delta_{ij} = \left\langle c_{i\downarrow} c_{j\uparrow} - c_{i\uparrow} c_{j\downarrow} \right\rangle_0$$

$$\chi_{ij} = \sum_{\sigma} \chi_{ij\sigma} \qquad m_i = \left\langle S_i^z \right\rangle_0$$

$$n_i = 1 - x_i \qquad n_{i\sigma} = \frac{1 - x_i}{2} + \sigma m_i$$

What are the periods of these collective excitations?

Fluctuating charge, spin, and pair modes:

$$x_i \rightarrow \bar{x} + \delta x_i \qquad m_i \rightarrow \bar{m} + \delta m_i \qquad \Delta_{ij} \rightarrow \bar{\Delta} + \delta\Delta_{ij}$$

In the case of $\boxed{\bar{m} = 0}$ and $\boxed{\bar{\Delta} \neq 0}$ (for h-doped cases),

The excitation will have c, s,p coupled modes due to

$$\sim \delta x_i \delta m_i^2 \rightarrow \delta x_q \delta m_{-q/2}^2 \qquad \sim \bar{\Delta} \delta x_i \delta\Delta_{ij} \rightarrow \bar{\Delta} \delta x_q \delta\Delta_{-q}$$

Collective mode pattern:

➔ Period: spin = 2 × charge ; pair = charge

Pair period

Charge period

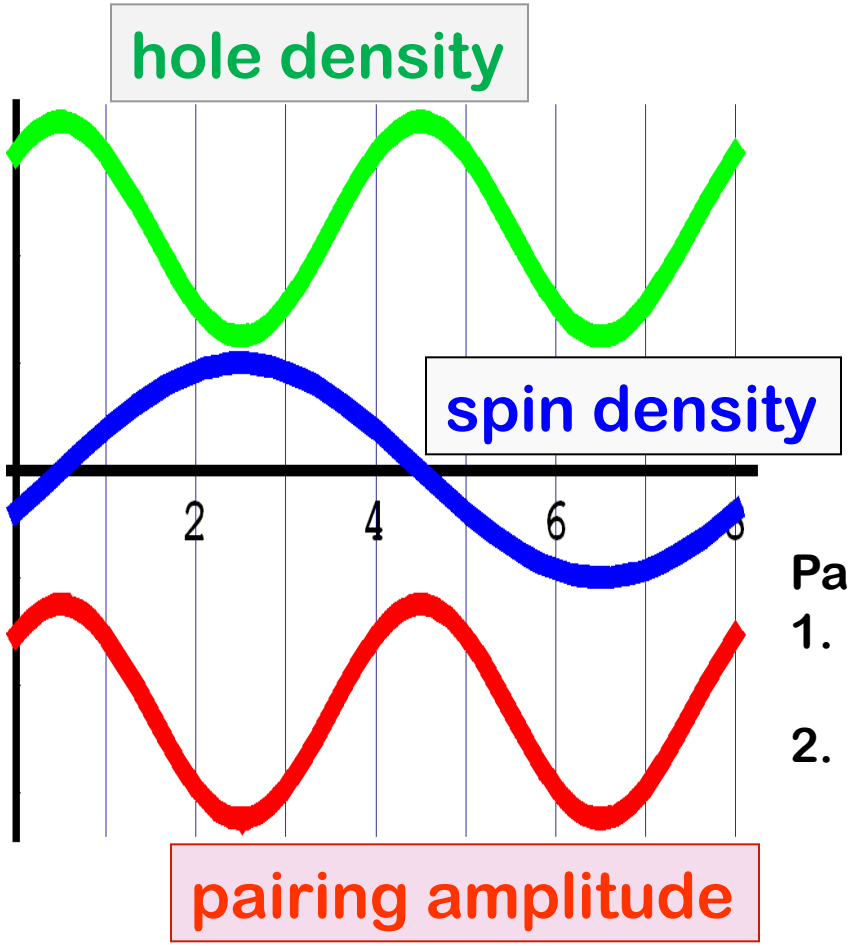
$$\boxed{a_C = a_P = \frac{a_S}{2}}$$

Spin peri

➔ Most favorable stripe pattern?

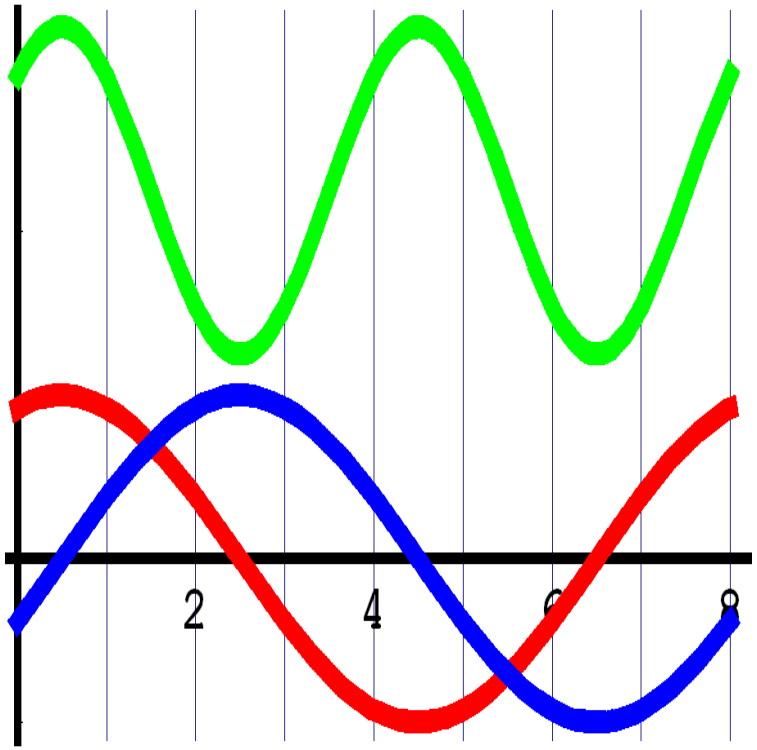
two different stripe patterns

AF-RVB Stripe
(Anti-Phase-m, In-Phase-Δ)



Pair field:
1. 2 × period
2. $\Delta^C = 0$

"AntiPhase" Stripe
(Anti-Phase-m, Anti-Phase-Δ)



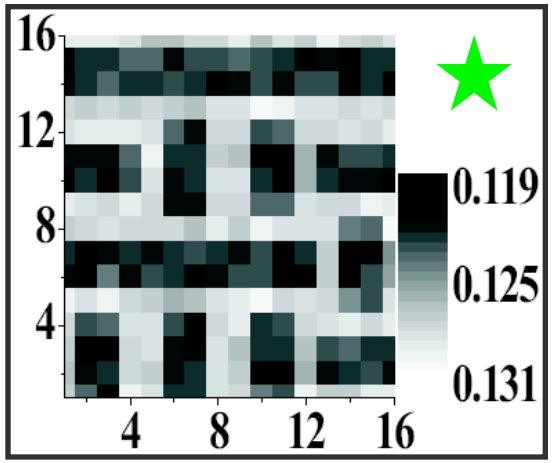
Himeda, *et al.*, PRL '02

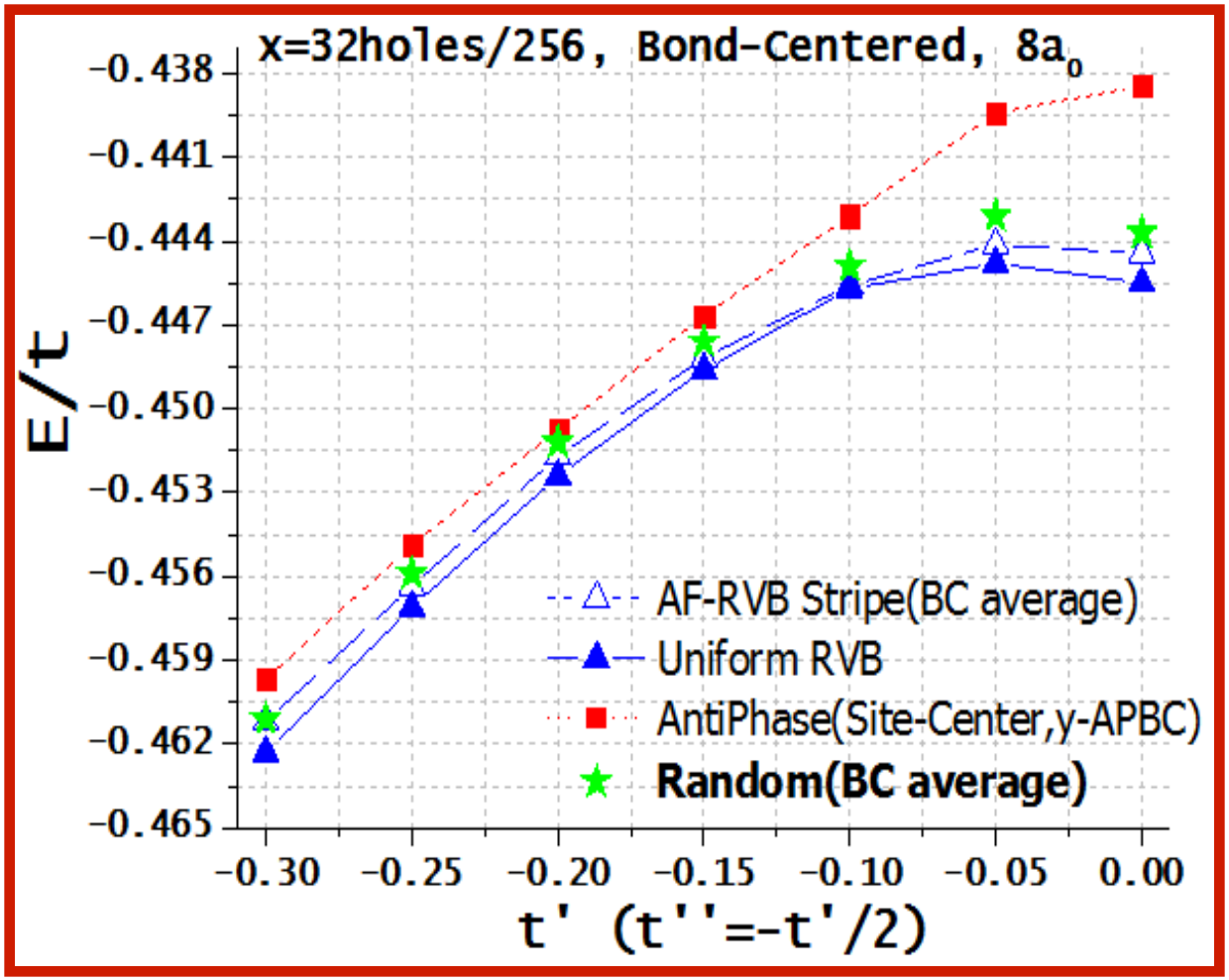
C.P. Chou, N. Fukushima, and T.K. Lee, PRB 78, 134530 (2008)

Optimized energy for $8a_0$ AR-RVB stripe

J/t=0.3

Random Pattern

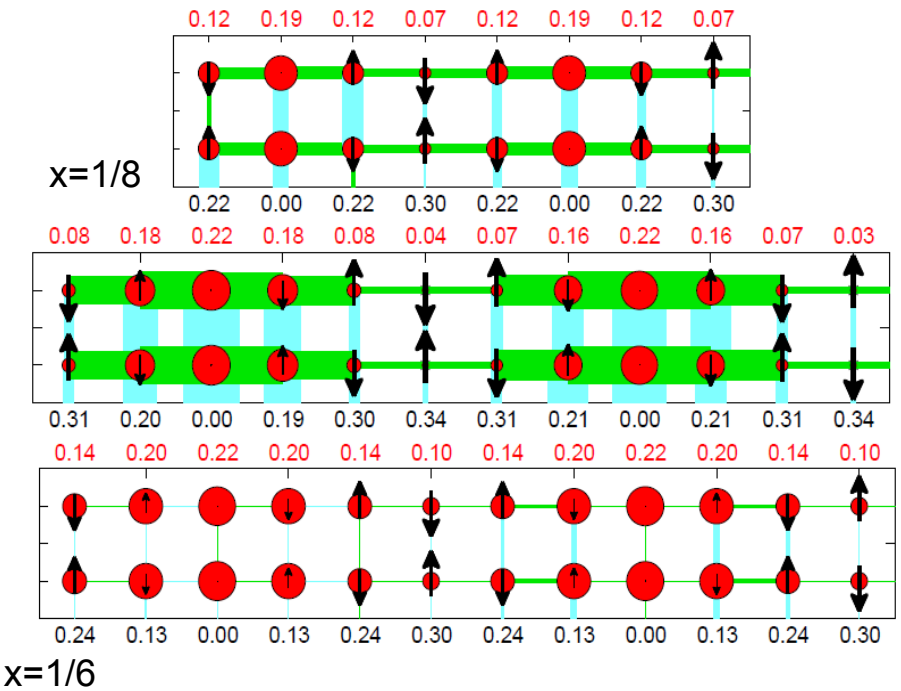




C.P. Chou, N. Fukushima, and T.K. Lee, PRB 78, 134530 (2008)

iPEPS also --- same type Stripe

iPEPS = infinite Projected Entanglement Pair State



Red number: hole density
Black number: magnetization
Bond size: pairing strength
(J/t=0.4)

The stripe they found in the pure t-J model:
same pattern as the AF-RVB stripe ➔ Period:
spin = 2 × charge ; pair = charge

The amplitude of modulation is a bit larger than our results.
These are their ground states.
For us, energies are too close to tell!

Corboz, *et al.*, PRB 84, 041108 (2011)

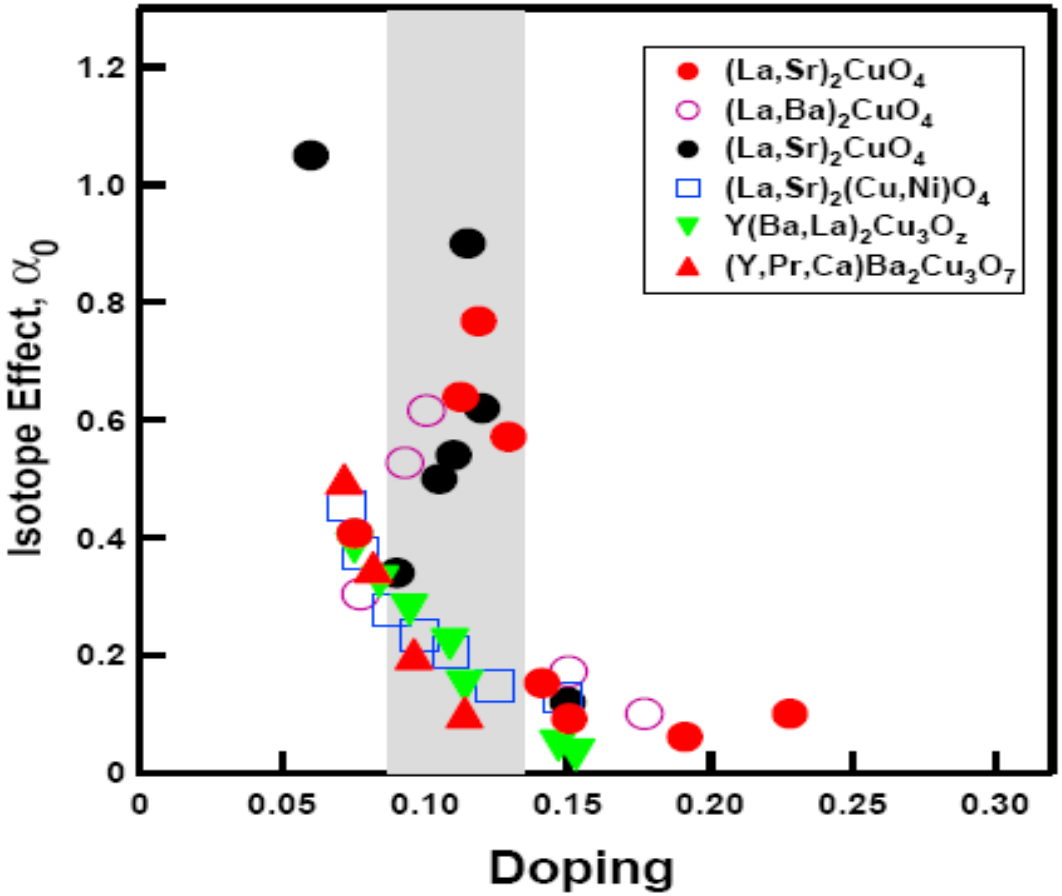
For extended t-J models, stripe states have very low energies, but may be not enough to replace the uniform ground states!

What extra microscopic ingredient is required to obtain stripes or half-doped stripes?

Electron-Phonon Interaction

Strong coupling of charge order to the lattice!

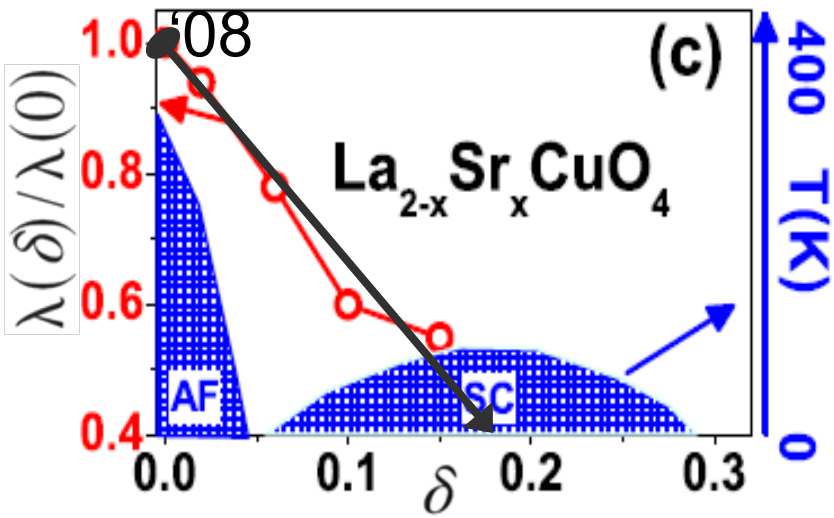
Isotope Effect



Zhou, et al., a chapter in "Treatise of High Temperature Superconductivity", ed. by J.R. Schrieffer '07

Mass-renormalization due to electron –phonon interaction

Mishchenko, et al.,PRL '08



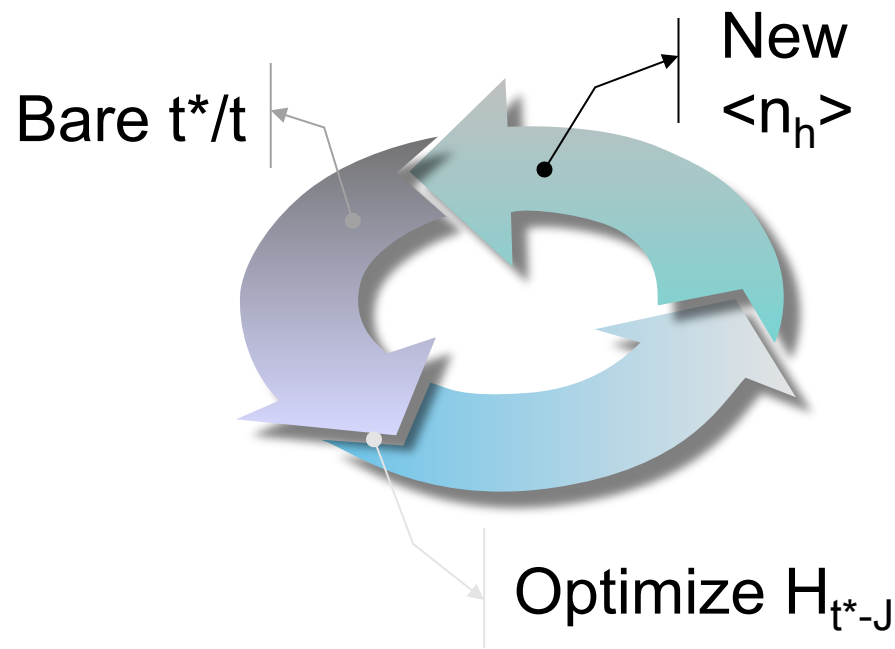
$$t^*_{ij} = t_{ij}\left\{1 - \Lambda\left[\frac{f(n_h^i) + f(n_h^j)}{2}\right]\right\}$$

$$\lambda(\delta)/\lambda(0) \equiv f(\delta) = 1 - 3\delta$$

$$\Lambda = \lambda(0)/\pi$$

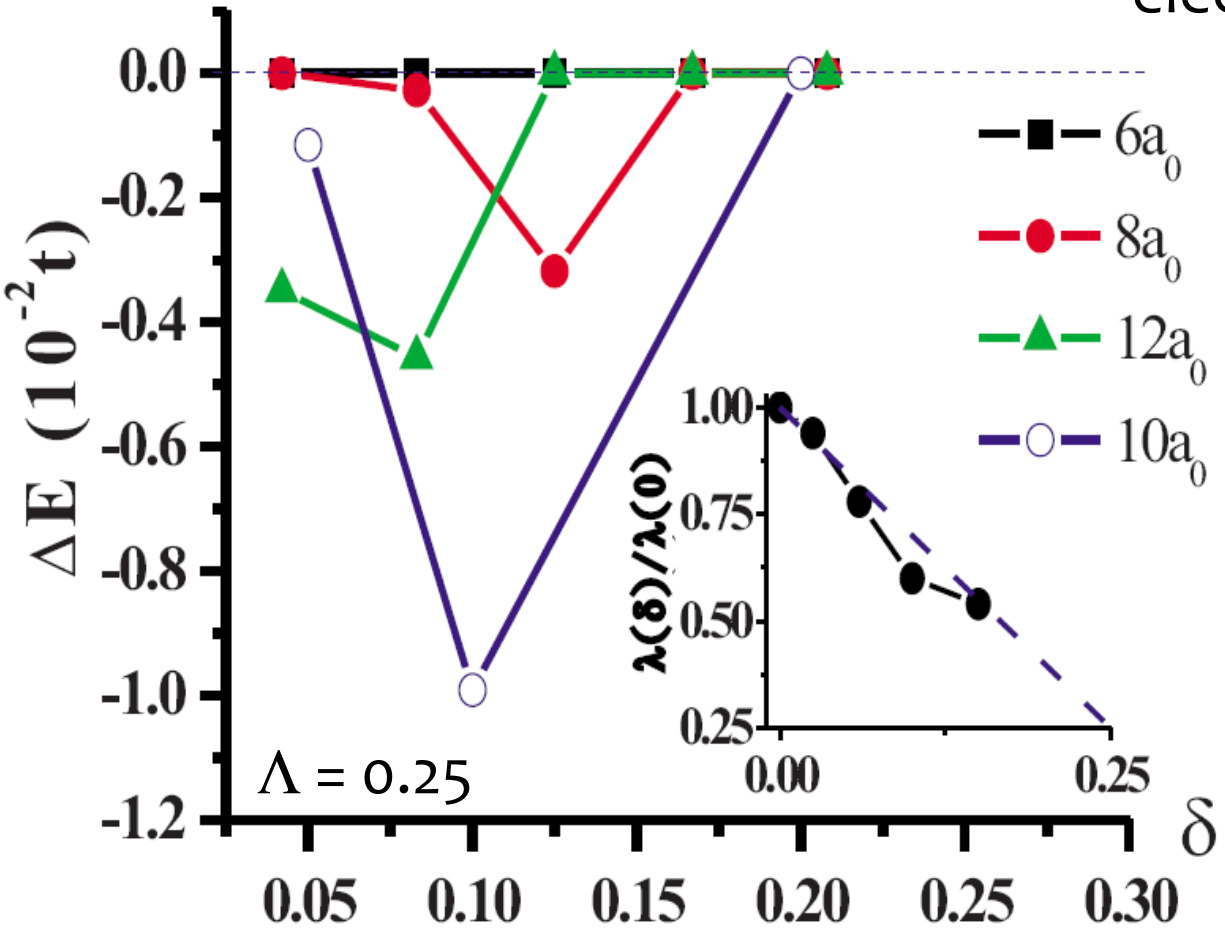
effective mass:

<u>Self-Consistent VMC</u>



Half-doped Stripe has been stabilized!

ΔE = E(Stripe)-E(RVB)



Mass renormalization by electron-phonon coupling

$$t_{ij}^{*} = t_{ij}\left\{1 - \Lambda\left[\frac{f(n_h^i) + f(n_h^j)}{2}\right]\right\}$$

$$\lambda(\delta)/\lambda(0) \equiv f(\delta) = 1 - 3\delta$$

$$\Lambda = \lambda(0)/\pi$$

$a_C \times \delta = 0.5$

$a_S \times \delta = 1.0$

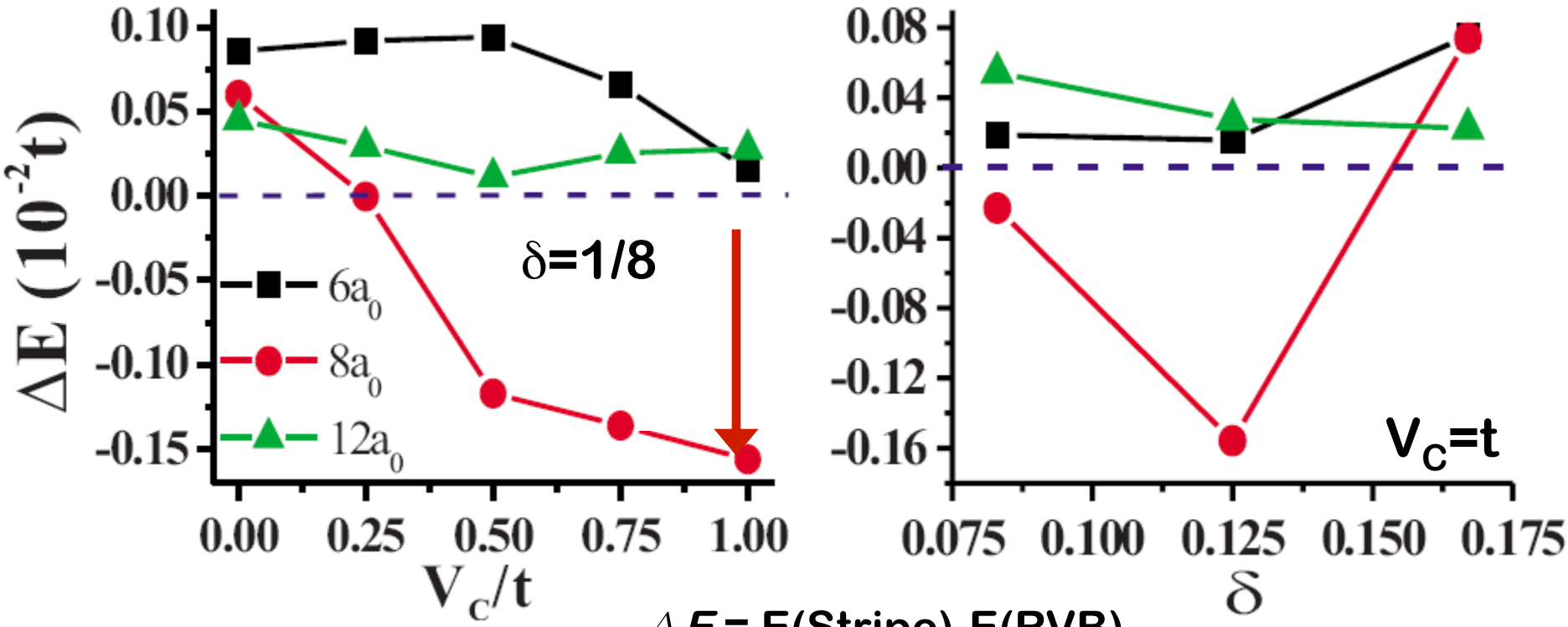
$(t',t'',J)/t = (-0.2,0.1,0.3)$

$\varepsilon_S = 1/a_S = \delta = \varepsilon_C/2 = 1/2a_C$

C.P. Chou and T.K. Lee, PRB 81, 060503(R) (2010)

Long range Coulomb – favors 8a stripe!

$$-\sum_{i,j,\sigma} t_{ij}\left(\tilde{c}^{\dagger}_{i,\sigma}\tilde{c}_{j,\sigma}+h.c.\right)+J\sum_{<i,j>}\vec{S}_i\bullet\vec{S}_j+V_C\sum_{i\neq j}\frac{\hat{n}^h_i\hat{n}^h_j}{\left|\vec{R}_i-\vec{R}_j\right|}$$



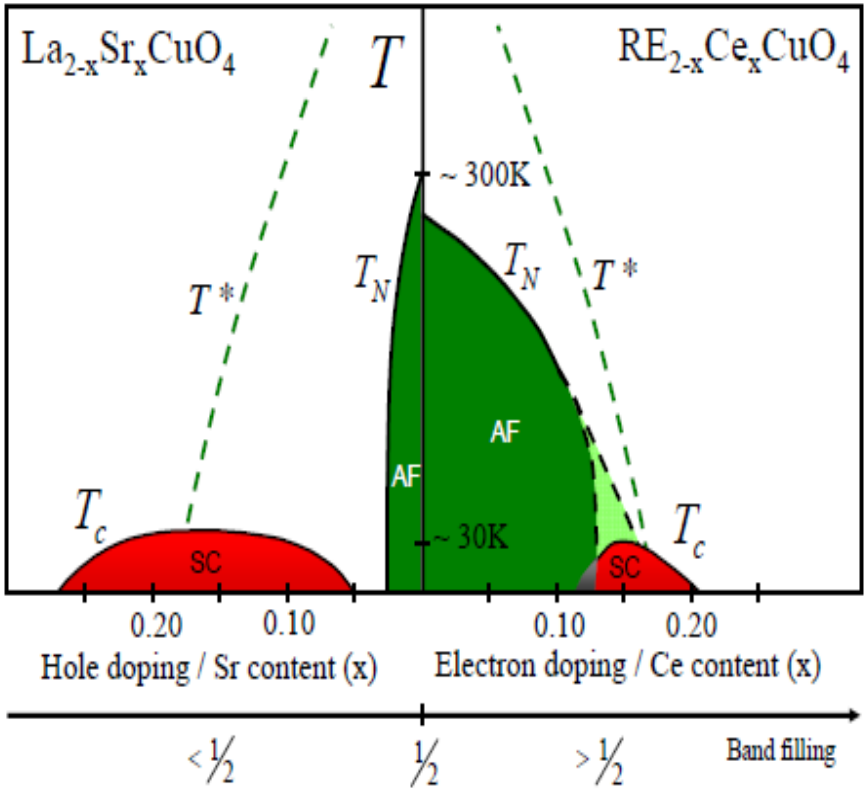
ΔE = E(Stripe)-E(RVB)

C.P. Chou and T.K. Lee, PRB 81, 060503(R) (2010)

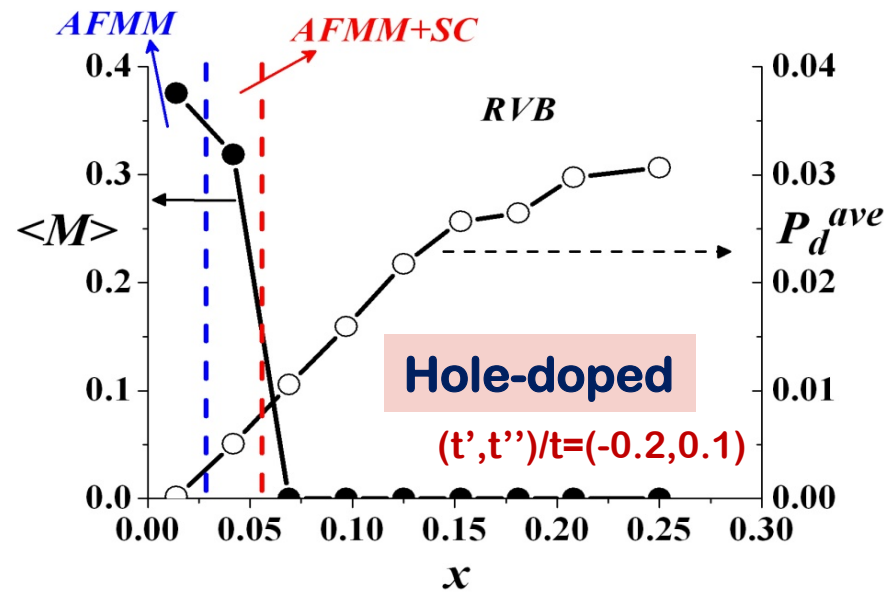
Electron-doped Cuprates

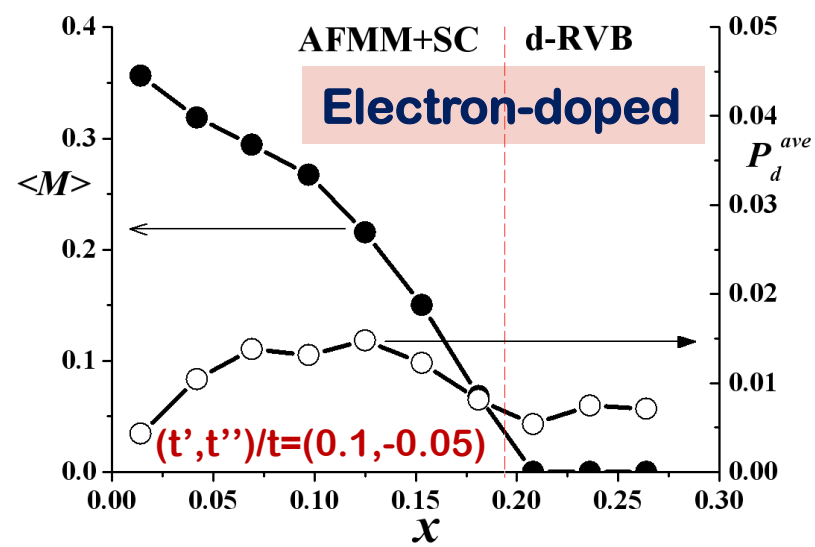
Same t-J model except t' and t'' have sign changed

Particle-hole Asymmetric!



Armitage, Fournier, and Greene, RMP '10





Inhomogeneous charge distribution ?

1. Many indirect experiments on the spatial inhomogeneity

   Bakharev, et al., PRL '04 ; Kang, et al., Nat. Mater. '07 ; Dai, et al., PRB '05 ; Zamborszky, et al., '04 ; Sun, et al., PRL '04 ; Kang, et al., PRB '05 …

2. A number of theoretical predictions (extended Hubbard model) on the filled or in-phase stripes

   Aichhorn, et al., EPL '05 ; Sadori, et al., PRL '00 ; Valenzuela, '06 …

3. There are also experimental evidences (chemical potential shift) against phase separation

   Niestemski, et al. Nature'07; Harima, et al., PRB '03 ; Yagi, et al., PRB '06 …

Up to now, no consensus yet!

What kind of period for stripe states?

Fluctuating charge, spin, and pair modes:

$$x_i \rightarrow \bar{x} + \delta x_i \qquad m_i \rightarrow \bar{m} + \delta m_i \qquad \Delta_{ij} \rightarrow \bar{\Delta} + \delta\Delta_{ij}$$

In the case of $\boxed{\bar{m} \neq 0}$ and $\boxed{\bar{\Delta} \neq 0}$ (for e-doped cases),

the significant contributions to the total energy are

$$\sim \bar{m}\delta x_i \delta m_i \qquad \sim \bar{\Delta}\delta x_i \delta\Delta_{ij}$$

Period: spin = charge = pair

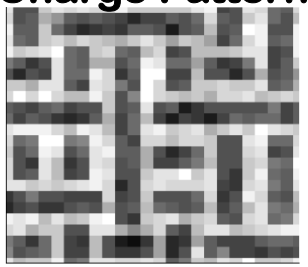
"In-Phase-m" stripe in electron-doped cases?

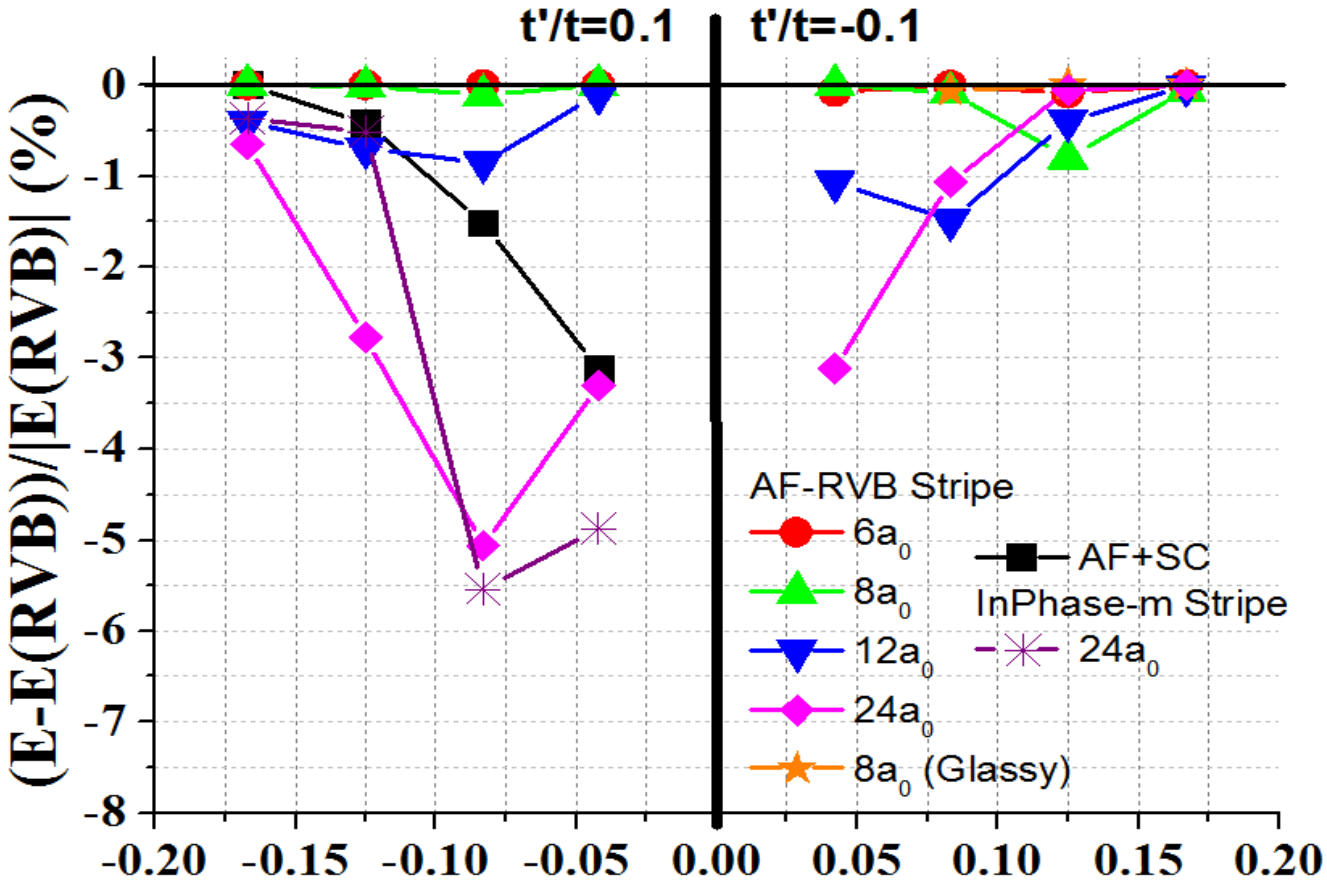
Phase Diagram (|t'/t|=0.1)

No half-doped stripe in electron-doped cases !

"phase separation" !?

Glassy Stripe Charge Pattern





24 × 24

(t'',J)/t=(-t'/2,0.3)

Electron doping ← → Hole doping

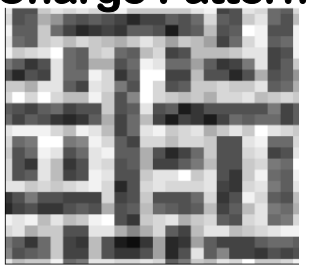
Λ = 0.25

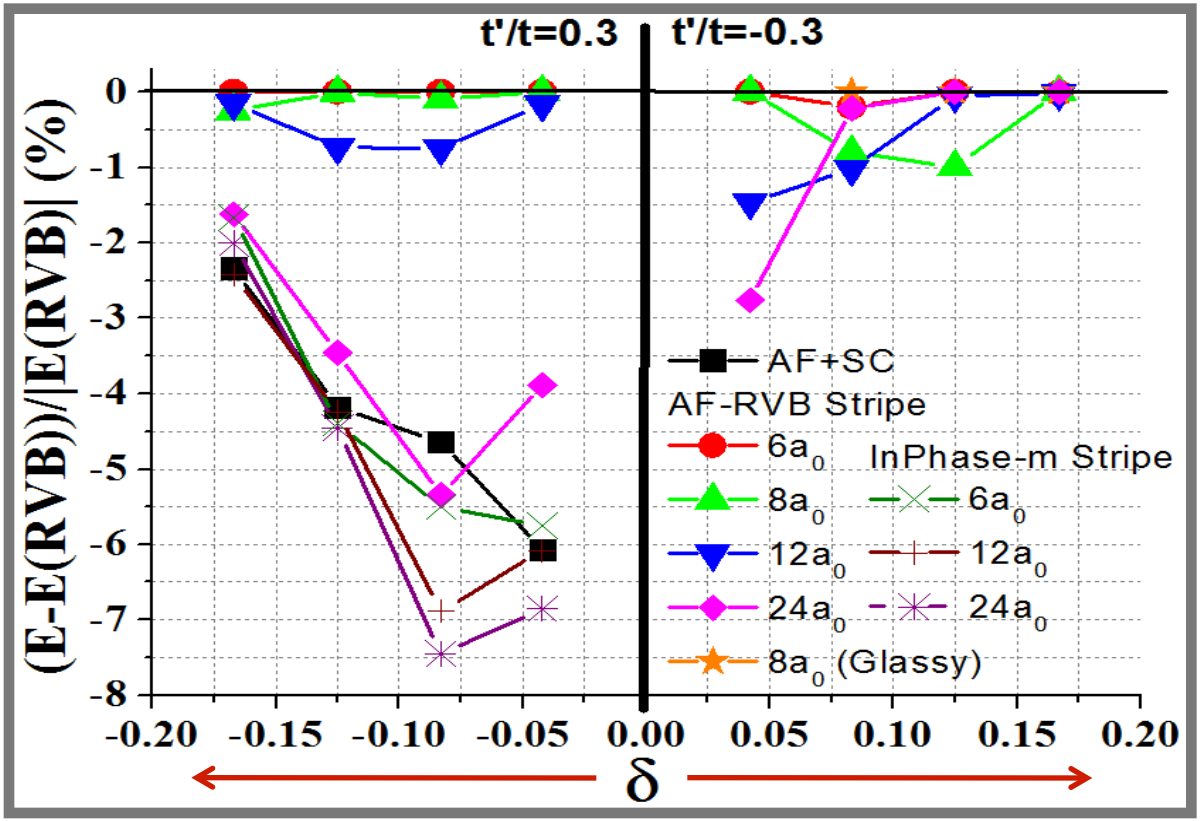
Phase Diagram (|t'/t|=0.3)

Stronger Particle-hole asymmetry !

Half-doped stripe for h-doped but not for e-doped !

Glassy Stripe Charge Pattern





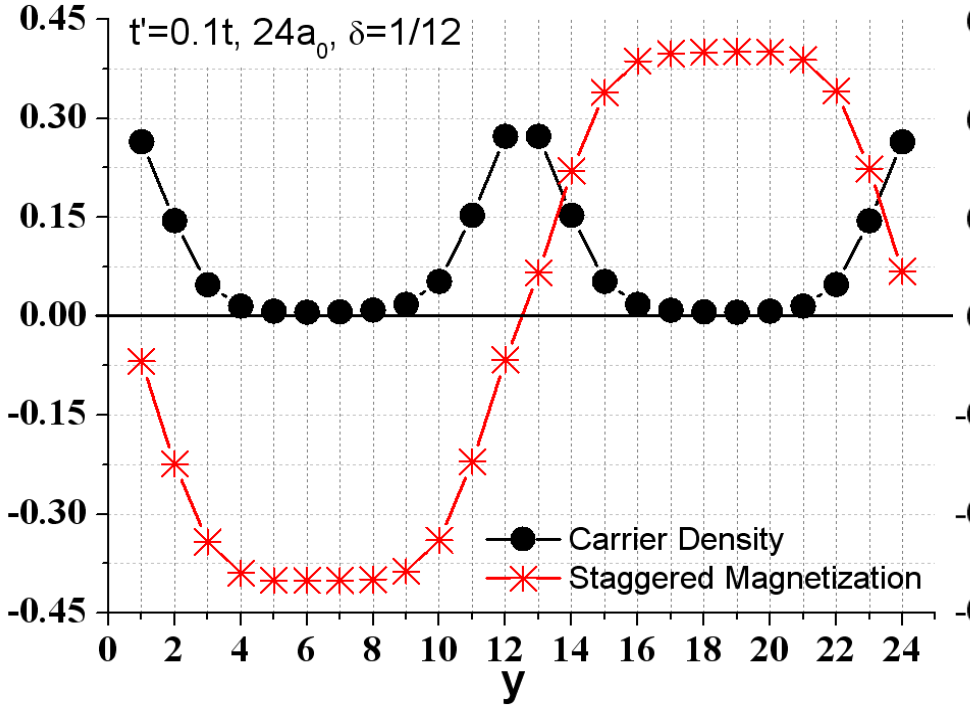
Electron doping **Hole doping**

Λ = 0.25

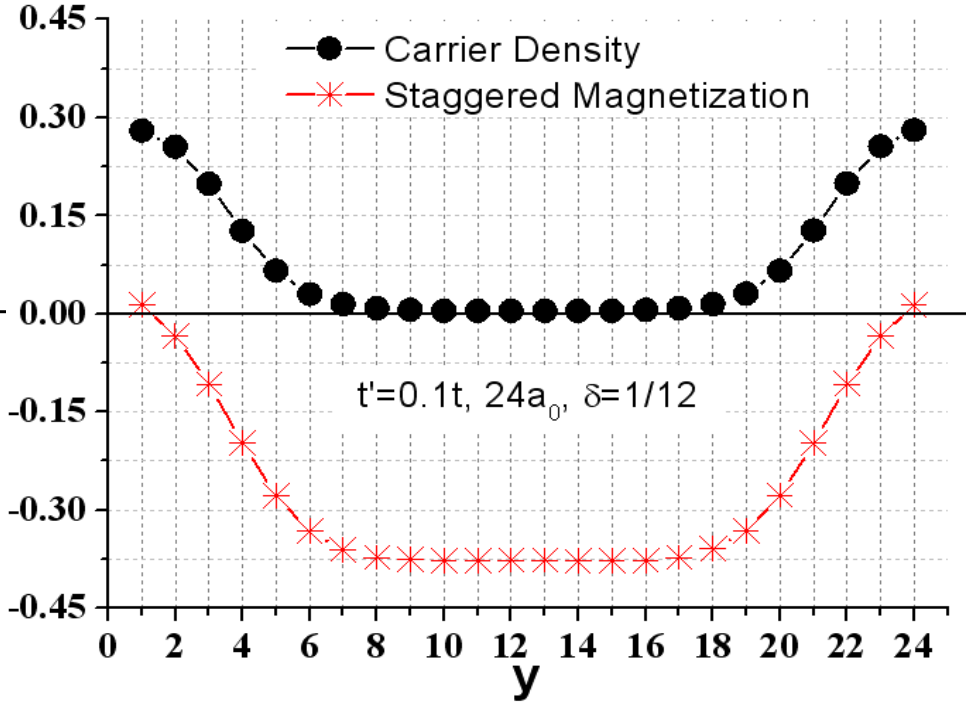
“Phase Separation” (t’/t=0.1)

Electron-rich phase v.s. Electron-poor phase (AF+SC state: |<M>| = 0.269)

AF-RVB Stripe



In-Phase-m Stripe



In-Phase-m Stripe: |<M>| = 0.265

Summaries

1. Collective excitations with spin, charge, and pair coupled naturally leads to appropriate lowest energy stripe patterns (AF-RVB) in the t-J-type model based on variational approach.
2. Long-range Coulomb interaction does not produce half-dopes stripes but it favors 8a at 1/8
3. A model to renormalize the mass due to electron-phonon interaction is proposed :
   - ✓ In the hole-doped regime,

     A weak electron-phonon interaction is enough to stabilize the half-doped vertical stripes for doping $\leq$ 1/8.

   - ✓ In the electron-doped regime,

     No stable half-doped stripe is found,

     phase separation seems more favorable.

Thank you for your attention!

Outline

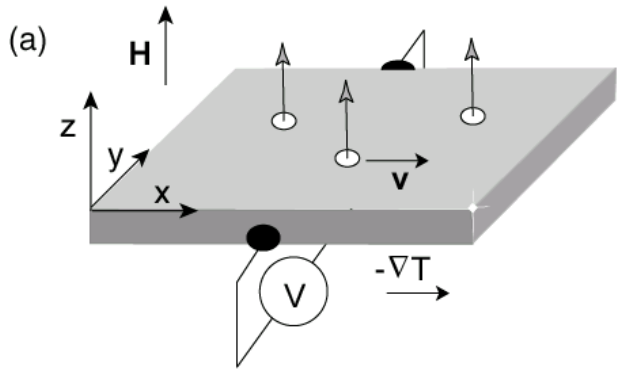
Use variational approach

Part I:
To study strongly correlated superconducting state with spatial inhomogeneity : a theory for stripes in high temperature superconductors

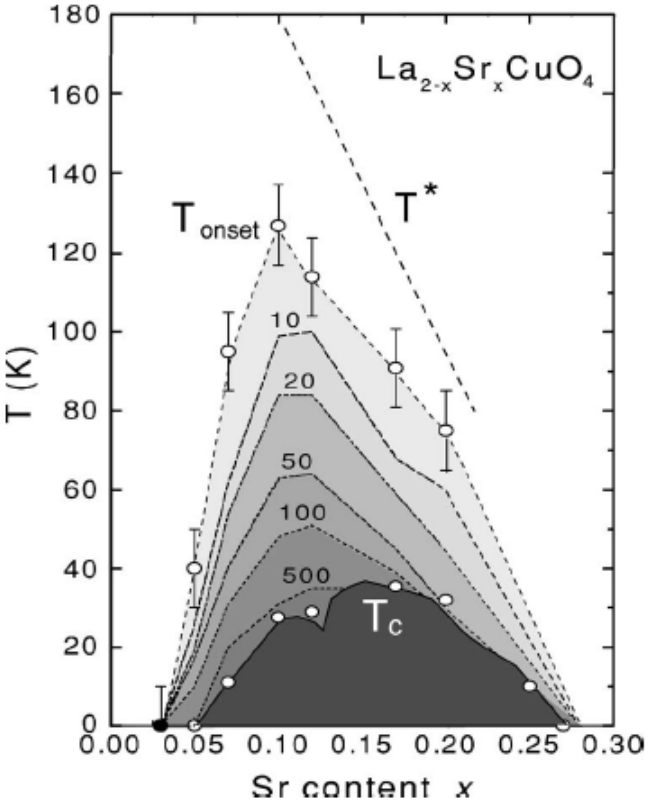
Part II:

To study phase fluctuation of the strongly correlated superconducting state

Giant Nernst effect and quantum diamagnetism



what caused this strong phase fluctuation?
competing interaction?



Emery and Kivelson, Nature, 1995
Strong phase fluctuation for cuprates, specially in the underdoped regime due to small number of holes,
is that enough?

Wang et al, PRB,2006

Until now, almost all of the variational calculations for the t-J model, except Yokoyama and Shiba (1988), have been using the canonical ensemble, a wave function with a fixed number of particles:

For N_e electrons $$|RVB\rangle = \mathrm{P_d}\left[\prod_k (u_k + v_k C^+_{k,\uparrow} C^+_{-k,\downarrow})\right]|0\rangle = \mathrm{P_d}|BCS\rangle$$

$$P_{N_e}|RVB\rangle = \mathrm{P_d}\left(\sum_k \frac{v_k}{u_k} C^+_{k,\uparrow} C^+_{-k,\downarrow}\right)^{N_e/2}|0\rangle = \mathrm{P_d}\left(\sum_{i,j} a_{i,j} C^+_{i,\uparrow} C^+_{j,\downarrow}\right)^{N_e/2}|0\rangle$$

What about the phase, the order parameter in SC?
Are the results same as using the wave function with varying particle numbers? Difference with BCS as there is this projection operator ?

number of particles fluctuates

$$\left|\Psi_{dRVB}\right\rangle = \hat{P}_d \prod_k \left(u_k + v_k c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow}\right)\left|0\right\rangle$$

Yokoyama and Shiba, JPSJ 1988

Particle-hole (P-H) transformation

$$c_{i\uparrow} \Longrightarrow f_i$$

$$c_{i\downarrow} \Longrightarrow d_i^{\dagger}$$

In df space, $\left|vac\right\rangle_{(df)}, d^{+}\left|vac\right\rangle_{(df)}, f^{+}\left|vac\right\rangle_{(df)}, f^{+}d^{+}\left|vac\right\rangle_{(df)}$

<u>Mapping Table:</u>

$$\left|h\right\rangle \Rightarrow \left|d\right\rangle_{(df)}$$

$$\left|\downarrow\right\rangle \Rightarrow \left|0\right\rangle_{(df)}$$

$$\left|\uparrow\right\rangle \Rightarrow \left|df\right\rangle_{(df)}$$

$$\boxed{\left|\uparrow\downarrow\right\rangle \Rightarrow \left|f\right\rangle_{(df)}}$$ ✖

($N_d + N_f = N$, if no magnetization)

Conserve "particles" **($N_d - N_f = N_h$)**

In the “df” representation

$$\left|\Psi_{dRVB}\right\rangle_{(df)} = \hat{P}_d \prod_k \left(u_k d_k^\dagger + v_k f_k^\dagger\right)\left|0\right\rangle_{(df)}$$

New process in the Monte-Carlo simulation

HOPPING:

$$\downarrow + \mathrm{h} \Leftrightarrow \mathrm{h} + \downarrow$$
$$0 + \mathrm{d} \Leftrightarrow \mathrm{d} + 0$$

$$\uparrow + \mathrm{h} \Leftrightarrow \mathrm{h} + \uparrow$$
$$\mathrm{df} + \mathrm{d} \Leftrightarrow \mathrm{d} + \mathrm{df}$$

EXCHANGE:

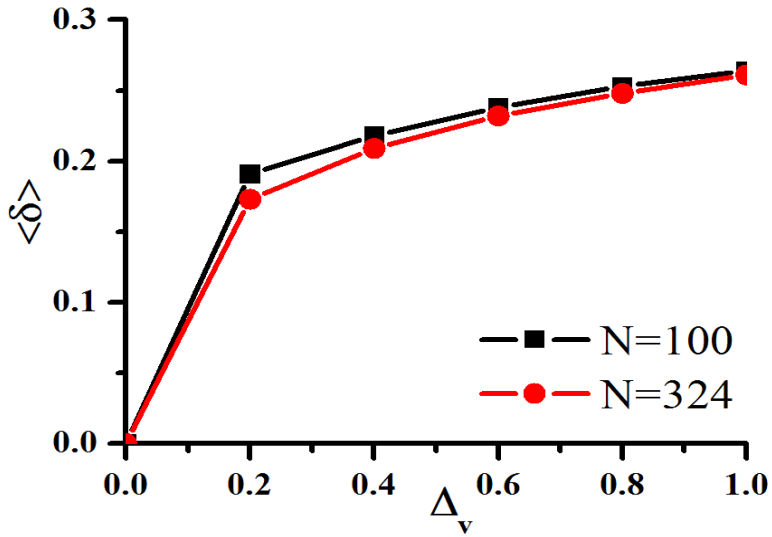
$$\uparrow + \downarrow \Leftrightarrow \downarrow + \uparrow$$
$$\mathrm{df} + 0 \Leftrightarrow 0 + \mathrm{df}$$

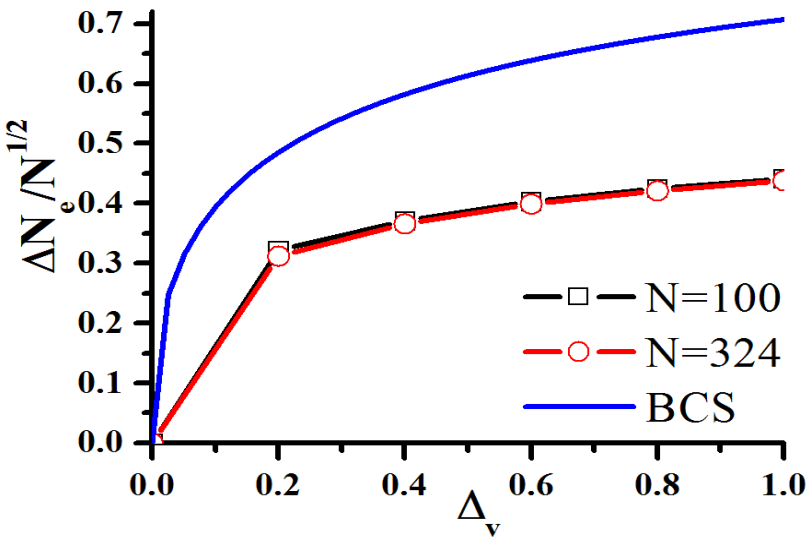
$$\uparrow + \downarrow \Leftrightarrow \mathrm{h} + \mathrm{h}$$
$$\mathrm{df} + 0 \Leftrightarrow \mathrm{d} + \mathrm{d}$$

Examine this wave function

$$\left|\Psi_{dRVB}\right\rangle_{(df)} = \hat{P}_G \prod_k \left(u_k d_k^\dagger + v_k f_k^\dagger\right)\left|0\right\rangle_{(df)}$$

it has a problem:





All variational parameters are fixed (to the half-filling case) except Δ_v

Average doping density <δ> becomes a function of the variational parameter Δ_v ! The projection operator changes the particle distribution function.

We must introduce a fugacity factor : g

$$\left|\Psi^g_{dRVB}\right\rangle_{(df)} = \hat{P}_d \prod_k \left(g u_k d_k^\dagger + v_k f_k^\dagger\right)\left|0\right\rangle_{(df)}$$

free-energy optimization

$$F\left(\mu_g\right) = E\left(N_e\right) - \mu_g N_e \qquad dF = 0 \;\Rightarrow\; \mu_g = \frac{\partial E}{\partial N_e}$$

Chemical potential μ_g is different from the variational parameter μ_v in BCS coherent factors!

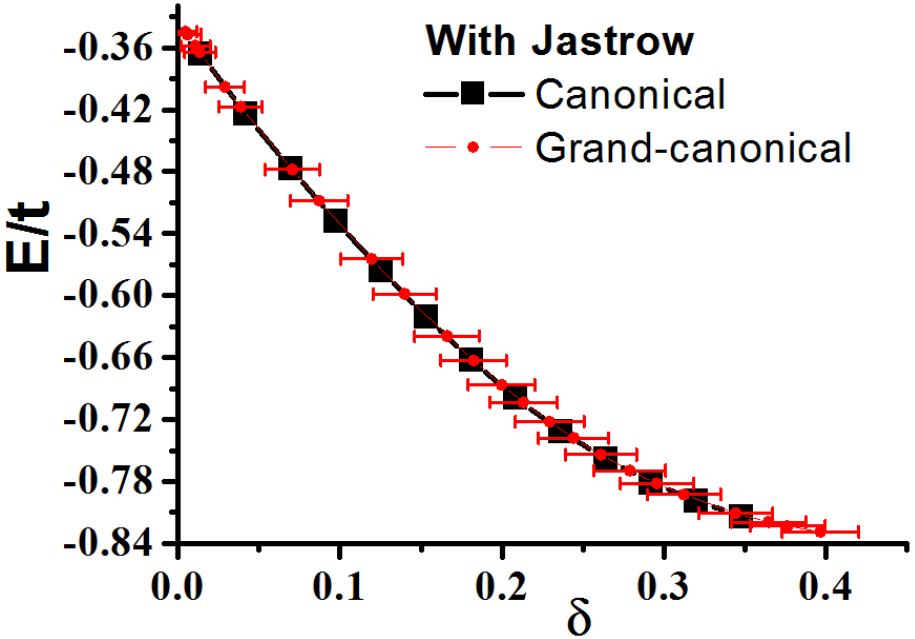
To optimize the variational energy, a hole-hole repulsive Jastrow factor is included!

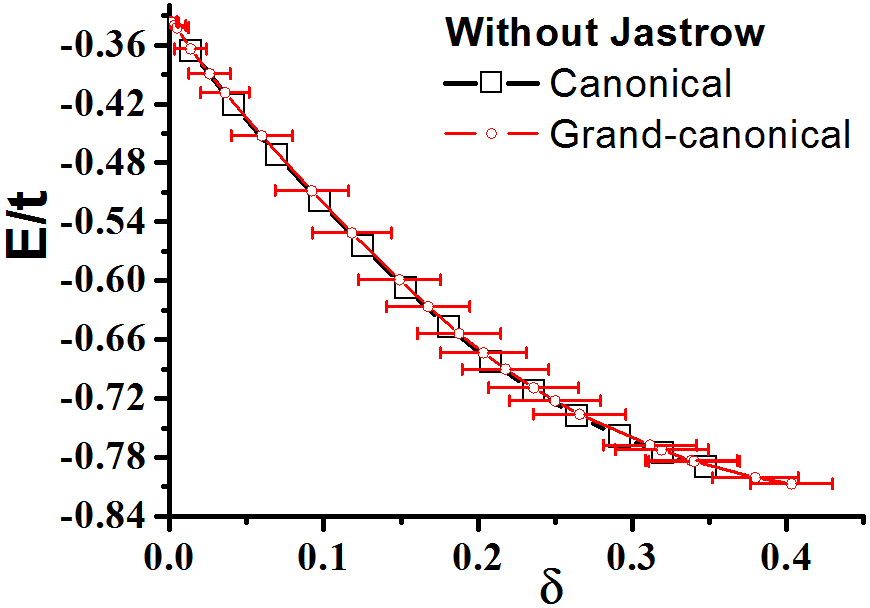
$$\hat{P}_J = \prod_{i<j} \left\{ 1 - \left[1 - \left|\vec{r}_{ij}\right|^{\alpha} \cdot \left(v_{\vec{\beta}}\right)^{\delta_{\vec{R}_j, \vec{R}_i + \vec{\beta}}} \right] \hat{n}^h_{\vec{R}_i} \hat{n}^h_{\vec{R}_j} \right\}$$

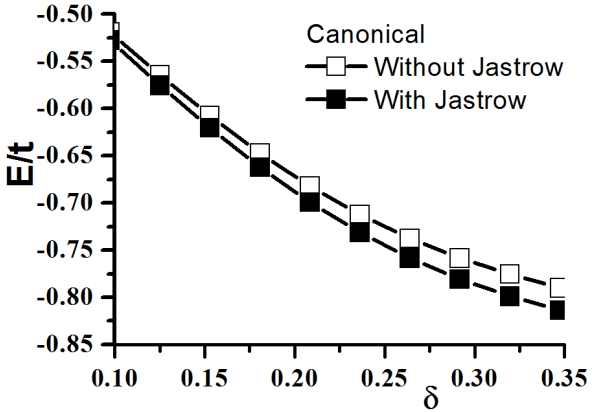
$\vec{r}_{ij} = \vec{R}_i - \vec{R}_j$ with PBC $\qquad \vec{\beta} = 1\text{st, 2nd, 3rd neighbors}$

Free Energy Minimization

consistency between the two ensembles







$$\left|\Psi^{g}_{dRVB}\right\rangle = \hat{P}_J \hat{P}_d \left|\Psi^{g}_{dBCS}\right\rangle$$

N=144

(t',t'',J)/t = (-0.3,0.15,0.3)

The phase fluctuation of d-RVB

$$\Delta\Theta = \sqrt{\frac{1}{2\Delta_0^2}\left(\left\langle \Psi^g_{dRVB}\middle|\hat{\Delta}^\dagger\hat{\Delta} - \hat{\Delta}^\dagger\hat{\Delta}^\dagger\middle|\Psi^g_{dRVB}\right\rangle\right)}$$

- **Three parts need to be calculated…**

$$\Delta_0 = \left|\sum_R \varphi_R \sum_i \left\langle c^\dagger_{i\uparrow} c^\dagger_{i+R\downarrow}\right\rangle\right| \qquad \varphi_{R=R_j-R_i} = \frac{1}{N}\sum_k \varphi(k) e^{ik(R_j-R_i)}$$

$$\varphi(k) = < c^+_{k\uparrow} c^+_{-k\downarrow} >$$

$$\left\langle \hat{\Delta}^\dagger\hat{\Delta}\right\rangle = \sum_{R_1,R_2\neq 0} \varphi_{R_1}\varphi^*_{R_2} \sum_{i,j} \left\langle c^\dagger_{i\uparrow} c^\dagger_{i+R_1\downarrow} c_{j\downarrow} c_{j+R_2\uparrow}\right\rangle$$

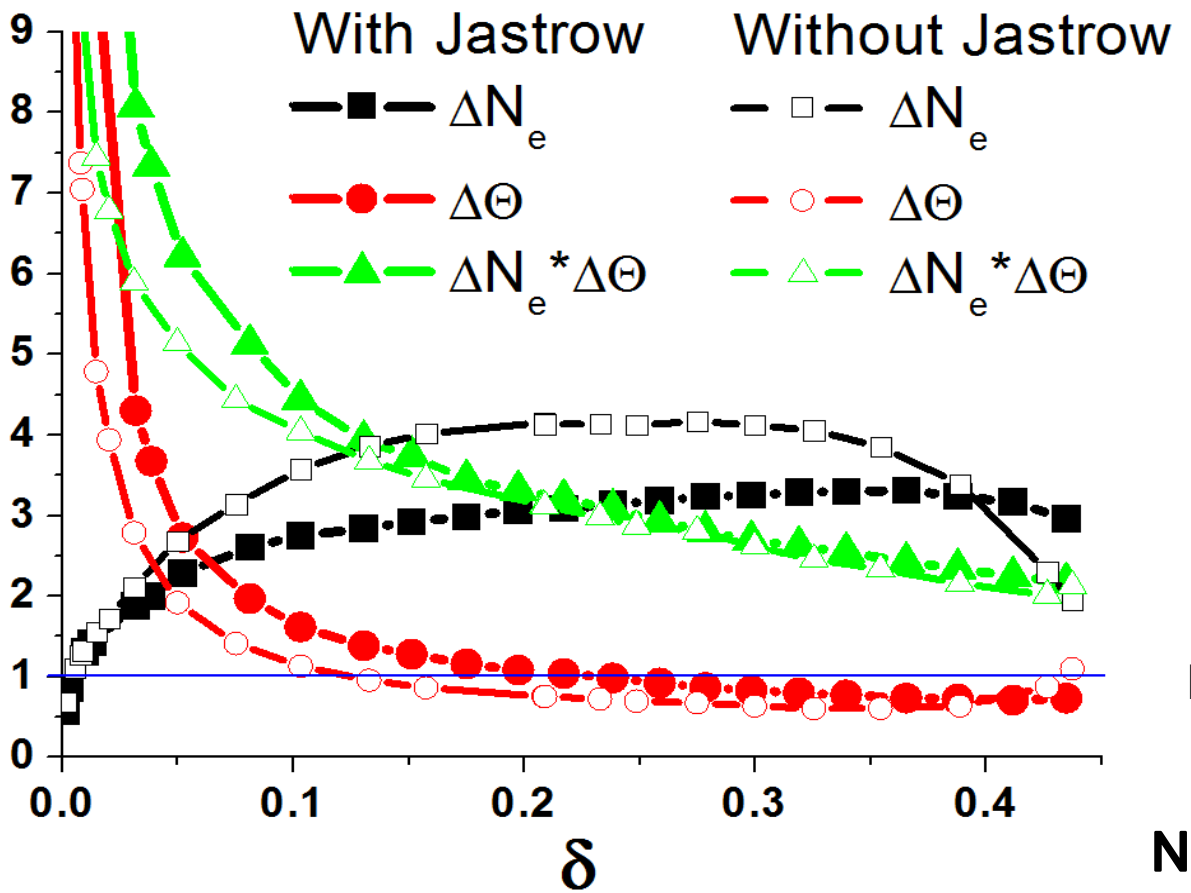
➔ **Pair-pair correlation!**

$$\left\langle \hat{\Delta}^\dagger\hat{\Delta}^\dagger\right\rangle = -\sum_{R_1,R_2\neq 0} \varphi_{R_1}\varphi_{R_2} \sum_{i\neq j} \left\langle c^\dagger_{i\uparrow} c^\dagger_{i+R_1\downarrow} c^\dagger_{j\downarrow} c^\dagger_{j+R_2\uparrow}\right\rangle \rightarrow \left\langle f^\dagger_i d_{i+R_1} f^\dagger_{j+R_2} d_j\right\rangle$$

➔ **Two hopping process!**

The doping dependence of fluctuations

Strong phase fluctuation near underdoped regions
➔ Away from the minimum uncertainty principle!



BCS value is 1

N=144
(t',t'',J)/t = (-0.2,0.1,0.3)

PRB 85, 054510 (2012).

Summaries

- An algorithm for the grand-canonical wave function is re-introduced.

- Projected BCS wave function is modified by a fugacity factor.

- Mott physics has produced a very large phase fluctuation, (Tesanovic) especially at UD for two main reasons
  1. very low charge carrier density
  2. the particle number distribution function is no longer a Gaussian. $\Delta\Theta\Delta N_e \gg 1$ --- instability of SC

  -- AFM, CPCDW (Cooper pair CDW), stripe, ..

- Chemical potential (and probably tunneling conductance) as a function of hole density agrees with experiments on cuprates fairly well.

Thank you for your attention!