The Statistical Mechanics of Financial Markets

Johannes Voit
Overview

1. Why statistical physicists care about financial markets
2. The standard model - its achievements and failures
3. Option pricing
4. Crashes
5. Introduction to risk measurement

On August 12, 2011 at Academia Sinica
“The Financial Crisis 2007-2009:
   How Did It Come? Will It Happen Again?”
Have You Heard of…

Physik-Neid
(German for „jealousy of physics“)

?
## Areas of Activity of Physicists in Financial Institutions

1. Risk management  
2. Quantitative modeling of financial engineering  
3. Trading  
4. Rating models  
5. Insurance and re-insurance  
6. Asset management, investment funds, hedge funds  
7. Power trading and modeling (electricity, gas, etc.)  
8. Consulting  
9. …  

Management positions
What does business management achieve?

- Why do banks exist? (Definitions and theorems, cooperation and competition, etc.)
- The loan
- The deposit
- Regulation
- Financial reporting
- Bank management
- Accounting
- Organization of banks
- Open questions

And FINANCIAL ENGINEERING???

And RISK MANAGEMENT???
What does the application of statistical physics to financial markets achieve?

- Understand and qualify geometric Brownian motion
- Fat tails and their scaling behaviors, non-Gaussian properties
- Temporal and interasset correlations
- Models and phenomenology for statistical properties of single assets
- Improvements in option pricing
- Microscopic market models (multiagent models)
- Episodic successes in the study of crashes
- …
There Is a Long History of Interaction between Physics and Finance

**Mathematics ↔ Economy**

- Phenomenology of idealized financial markets
  - Price fluctuations are Gaussian random variables
  - Subsequent fluctuations are independent
- Game theory approach → equilibrium

**Physics ↔ Economy**

<table>
<thead>
<tr>
<th>Phenomenon</th>
<th>Physics</th>
<th>Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random Walk</td>
<td>Einstein (1905)</td>
<td>Bachelier (1900)</td>
</tr>
<tr>
<td>Fractals</td>
<td>Mandelbrot (1975)</td>
<td>Mandelbrot (1963)</td>
</tr>
<tr>
<td>Random matrix theory</td>
<td>Wigner (1951)</td>
<td>Laloux et al., Plerou et al. (1997)</td>
</tr>
</tbody>
</table>
What Are the Objects of Our Studies? (I)

**Assets** (securities, equities)
- Value depends on economic factors, company profits, interest rates, market conditions, “sentiment”, …
- Examples: stocks, currencies, commodities (gold, oil, wheat, pork bellies, electrical current, …)
- Important in practice but seldom covered by physicists: bonds
- Assimilated market indices, e.g., S&P 500, Dow Jones Industrial Average, DAX, Taiwan Weighted, …
- Traded
  - at organized exchanges, e.g. NYSE, NASDAQ, Taiwan Stock Exchange, …
  - Dark pools
DAX – The German Blue Chip Stock Index

- Statistics of price changes: continuous? discontinuous? Frequent/rare jumps?
- What causes price changes on various time scales?
- Importance of economic factors?
- Profitable investment during upward AND downward moves?
- Implications for derivatives (futures, options)?
- Crashes predictable?
Long Time Scales Are Not So Different

Conclusions on profitability of investments may change in time:

- $t \leq 2000$: long-term investments profitable!
- $t > 2000$: ???
Questions Which Drive Physicists in Their Venture into Financial Market Research

Can we improve our understanding of financial markets using analogies to phenomena in nature, such as

• Diffusion
• Driven systems, nonlinear systems, chaos
• Formation of avalanches
• Earthquakes
• Phase transitions
• Turbulent flows
• Fractals
• Highly excited nuclei
• Electronic glasses, etc.
What Are the Objects of Our Studies? (II)

Derivatives

• A security whose value depends on other, more basic underlying variables …
• Often: … the value of another security (“the underlying”), e.g., stocks, bonds, market indices, currencies
• But also: commodities, market volatility, weather, …
• Traded
  • at organized exchanges, e.g. CBOT, CME, EUREX, LIFFE, if products are standardized
  • OTC = over the counter (financial institution ↔ corporate client), tailored to individual needs of the client
Three Important Derivatives (I)

1. Forward (Contract)
   - Delivery of an asset at time $T$ in the future for a price $F$ fixed today
   - Binding contract without choice for either party, usually OTC

2. Futures (Contract)
   - Delivery of an asset at time $T$ in the future for a price $F$ fixed today
   - Binding contract without choice for either party
   - Standardized product, usually traded at organized exchanges
   • Effect: lock in price of delivery
   • Profit of a long position: $S(T) - F$
     - $F$ … agreed delivery price, $S$ … spot price
Three Important Derivatives (II)

3. Options (call / put)
   • Gives the holder the right to
     • buy (call)
     • sell (put)
     an asset at time T in the future for a price X (… strike price)
   • Underwriter is obliged to deliver
   • Traded at exchanges or OTC
   • Insurance contract
   • What is the fair price c (or p)? Asymmetric profile of choices makes valuation nontrivial
Three Important Derivatives (III)

Payoff profile of options at maturity

\( c \) … option price
\( X \) … strike price
\( S_T \) … spot price at \( T \)
References