The Statistical Mechanics of Financial Markets

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Overview

1. Why statistical physicists care about financial markets
2. The standard model - its achievements and failures
3. Option pricing
4. Crashes
5. Introduction to risk measurement

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What does business management achieve?

• Why do banks exist? (Definitions and theorems, cooperation and competition, etc.)
• The loan
• The deposit
• Regulation
• Financial reporting
• Bank management
• Accounting
• Organization of banks
• Open questions

And RISK MANAGEMENT???
What Is Risk?

• resecum (lat.), ριξικον (gr.) = cliff
• Something unpleasant happening?
• Hazard, a chance of bad consequence, loss or exposure to mischance (Oxford dictionary)
• Any event or action that may adversely affect an organization’s ability to achieve its objectives and execute its strategies (McNeil, Frey, Embrechts)
• Deviation of a specified quantile of a (profit-and-) loss distribution from its expectation value (Ali Samad-Khan)
  – NB: this statement apparently defines risk through its measurement process!
Why Risk Measurement?

• “You only can manage what you measure”
• Determination of risk capital and capital adequacy
  → banking regulation
  → economic capital
  – The amount of capital shareholders should invest in a company in order to limit its probability of default to a certain confidence level
• Management tool
  → Basis for limit setting
• Insurance premiums
  → compensate insurance for bearing the risk of claims
The Role of Capital As a Buffer against Risk

- **Regulatory capital** (→ banking regulation)
  - Is the amount of capital the supervisors require a bank to hold
- **Economic capital** (applies to both banks and non-bank corporates)
  - (theoretically) Is the amount of capital a bank's shareholders would choose to cover its risk / ensure continuous operation in the absence of external regulation
  - BIS: capital which a bank holds based on its own assessment of risk
  - Gives full benefit of risk diversification
- **What is capital?**
  - Capital needed → risk measurement
  - Capital available
    - Regulatory: recognized capital constituents
    - Economic: value of all assets
What Risks Faces a Bank? (And Any Other Corporation, And Any Other Individual)

- Market risk
- Counterparty default risk
- Liquidity risk
Defining the essential risks

• **Market risk** is the risk of loss of a position in a security, portfolio, etc. due to changes in market conditions.

• **Credit risk** is the risk of loss due to a counterparty in a financial contract not satisfying her contractual obligations.

• **Operational risk** is the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events. Includes legal risk. Excludes strategic and reputational risk.

• **Liquidity risk** is the risk that an asset cannot be traded fast enough to prevent a loss, resp. remain solvent (insolvency risk). Refunding risk … increased cost of refunding due to market illiquidity.
How To Measure Risk?

• Notional amount
  – Sum of notional values of securities
  – When modified by “risk weights”, often used by regulators, e.g. standardized approach in Basel framework for market and credit risk
  – Neglects effects of hedging
  – Neglects the effects of diversification

• Variance / standard deviation = volatility
  – Second moment must exist
  – Only symmetric distributions
  – Convergence properties under fat-tailed distributions
Value at risk is the most popular risk measure (I)

- $P(L \leq l)$ … probability that a loss $L$ is below a certain value $l$
- Idea: maximum loss not exceeded with a given (high) probability
  \[ \text{VaR}_\alpha = \inf_l \left\{ P(L > l) \leq 1 - \alpha \right\} = \inf_l \left\{ P(L \leq l) \geq \alpha \right\} \]
- Quantile of loss distribution
- Typically, $\alpha = 0.95$ or $0.99$

Source: McNeil, Frey, Embrechts
On notation

• “The Statistical Mechanics of Financial Markets” uses log-returns over specified time scale $\tau$
• (Spot) price of an asset at time $t$: $S(t)$
• Continuous compounding with rate $\mu$: $S(t+\tau) = S(t) \exp(\mu \tau)$

• Log-return $\delta S_{\tau}(t) = \ln \frac{S(t)}{S(t-\tau)}$

• Frequent proxy for forward-looking risk measures

$$\delta S'_{\tau}(t + \tau) = \ln \frac{S(t + \tau)}{S(t)} \approx \ln \frac{S(t)}{S(t-\tau)} = \delta S_{\tau}(t)$$

• Translational invariance in time assumed, “future $\approx$ past”
Value at risk is the most popular risk measure (II)

- \( L = - \delta S_\tau(t+\tau) = - \delta S_\tau(t) \) [when \( \delta S_\tau(t+\tau) \) negative, stationary] contains a time scale \( \tau \), dependent on scale of main business
  - \( \tau = 1 \) day for trading limits
  - \( \tau = 10 \) day for market risk management
  - \( \tau = 1 \) year for credit and operational risk management
- Definition of VaR is a practical working definition, accurate mathematical definition can be given
- When VaR is calculated on bank level, \( \alpha \) is related to default probability of bank, and therefore to its rating
- Conversely, a given target rating determines \( \alpha \)
- \( VaR_\alpha - \langle L \rangle \) is a related risk measure
  - Sometimes called value at risk, mean-VaR, unexpected losses
Default probabilities determine rating scores

- Default probabilities (PD) translate into confidence levels $\alpha$ for risk measurement: $PD = 1 - \alpha$

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<th>Implied PD</th>
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<tr>
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<td>Aa1</td>
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<td>AA</td>
<td>Aa2</td>
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<td>A</td>
<td>A2</td>
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<tr>
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<td>BBB</td>
<td>Baa2/Baa3</td>
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<td>BB+</td>
<td>(Ba1)</td>
<td>0.6% (0.9%)</td>
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<tr>
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<td>Ba2</td>
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- Investment grade
- Junk bonds
Pros and cons of VaR as a risk measure

+ Implements managerial view: clear-cut separation of what can be managed (events below confidence level) and of what cannot be managed (events above confidence level)

+ Recognized by regulators (cf. below, Basel framework for market risk)

- VaR is not subadditive:
  - Assume a bank with two portfolios with loss variables $L_1$ and $L_2$, and VaR$_1$ and VaR$_2$ at the same confidence level $\alpha$
  - Loss of bank is $L = L_1 + L_2$
  - Then $\text{VaR}_{1+2} \leq \text{VaR}_1 + \text{VaR}_2$ (subadditivity property)
    IS NOT NECESSARILY SATISFIED
Examples for the failure of VaR

• Short position in far-out-of-the-money call and put options

  - 4% loss probability from put
  - 4% loss probability from call

  \[ p(S) \]

• No risk at 95% confidence level in each separate position
• However, significant risk to combined position
• Failure of VaR observed in many other instances
Coherent Risk Measures

• A coherent risk measure $\rho(X)$ satisfies the following four properties
  (X,Y ... values of positions, i.e. “risk” comes from negative X,Y)

• Subadditivity: $\rho(X+Y) \leq \rho(X) + \rho(Y)$
  [ $\rho(X+Y) = \rho(X) + \rho(Y) \iff X$ and $Y$ perfectly correlated ]

• Translation invariance (risk-free condition): $\rho(X+rn) = \rho(X) – n$
  r ... risk-free interest rate

• Positive homogeneity of degree 1: $\rho(\lambda X) = \lambda \rho(X)$

• Monotonicity: $\rho(X) \leq \rho(Y)$ if $X \geq Y$
Expected shortfall is a coherent risk measure

- Expected shortfall \( ES_\alpha = \frac{1}{1 - \alpha} \int_0^\infty L p(L) dL \) \( \geq VaR_\alpha \)

- Simplified definition for integrable loss variables with continuous distributions, accurate definition can be given
- VaR just controls probability of bad event, not its consequences
There Is a Big Gap in Risk Measurement

- Aggregation of individual securities to portfolio risk measurement mainly by Monte Carlo simulation
  - Alternative 1: historical simulation
  - Alternative 2: variance-covariance model (Gaussian world, $\text{VaR} \propto \sigma$, prefactor dependent on confidence level)

$$\text{VaR}_{tot} = \sqrt{\text{VaR}_1^2 + 2 \rho_{12} \text{VaR}_1 \text{VaR}_2 + \text{VaR}_2^2}$$

- Systematic aggregation from portfolio level to bank level almost not feasible (copulas)
References
